CHAPTER 5

The sampling process

Digital seismograms are sequences of numbers which in general have been obtained from the continuous output voltage of a seismic sensor by the procedures of sampling and analog to digital conversion. We will now consider some of the characteristic properties of these processes. We will simulate the sampling of analog data and its reconstruction from sampled values. We will find that an analog signal can only be reconstructed from its sampled values, if the frequency content of the signal to be sampled contains no energy at and above half of the sampling frequency (sampling theorem). We will investigate what happens if we deliberately violate this rule (aliasing effect).

5.1 The sampling of analog data

When we use a computer program to model continuous phenomena - like simulating the output voltage of a seismometer for certain boundary conditions - we usually do not think about the underlying process. We just do it and assume the results are meaningful. In terms of system theory, however, we made an important transition: From a continuous system to a discrete system. This step entails some rules we can not violate without jeopardizing the results.

The same transition from a continuous system to a discrete system takes place when we acquire data in digital form. In this transition there are actually two different steps:

- **Sampling or discretization** — Taking discrete samples of a continuous data stream. The data may still be in analog representation after the sampling process.

- **Analog to digital conversion (quantization and coding)** — For voltage signals, this steps normally occurs in an electronic device which is called ADC, 'analog to digital converter'. After having gone through this step, which we will treat in chapter 6, the data are digital and discrete.

In Fig. 5.1 the principle of the discretization process is displayed schematically. A continuous signal is sampled at discrete times indicated by the positions of the vertical arrows. The amplitudes of the arrows correspond to the signal amplitudes at the sampling
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times. We can also view the discretization process as modulating a sequence of delta impulses which are separated in time by \( T \). In the present context, only constant sampling intervals \( T \) (equidistant discretization) are relevant. Its reciprocal, \( 1/T = f_{\text{dig}} \), is called the \textit{sampling frequency} or the \textit{digitization frequency}.

![Fig. 5.1 Sketch of the discretization (sampling) process. The vertical arrows show the locations and the values of the samples. \( T \) denotes the sampling interval.](image)

Below, some of the effects of sampling of 'continuous' signals are demonstrated using the \textit{discretization tool} within DST. Of course, all the data traces in DST are already in digital form that is in form of a sequence of numbers. We are only approximating a continuous signal by one which has been sampled at a sampling frequency much higher than the one at which we want to investigate. Let us assume for now without proof that this is a valid approximation. From the information in this chapter it should become clear under which conditions this is permissible. Also, let us assume without proof that we can reverse the sampling process and recover an approximation of the original continuous signal. In DST, a procedure called \textit{Whittaker reconstruction} is used for this task. For the details of this process, which we do not need to be concerned about for the purpose of the following argument, see for example Stearns (1975).

Fig. 5.2 displays a sinusoidal input signal with a signal frequency of 1 Hz which we will use for simulating the discretization process with DST.