A Process Application of Step-Wise Safe Switching Control

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Abstract- In the present work we study the application to a characteristic nonlinear process of the Step-Wise Safe Switching (SWSS) control approach for SISO plants. The process nonlinear description is considered to be unknown for control design, while its behavior is approximated by a set of SISO linear models derived through identification around corresponding operating points. The performance of the supervisory scheme is illustrated using simulation results, which are also compared with corresponding results of other approaches.

I. INTRODUCTION

Supervisory switching control for nonlinear processes (see for example [1]-[8]) comprises, in general, a set of field controllers and a supervisory controller. Each field controller achieves the required closed-loop performance provided that the plant’s input/output (I/O) trajectories remain within a limited range of operation. The supervisory controller implements switching between the appropriate field controllers, as the plant’s I/O trajectories move between different areas of operation.

A switching approach, called Step-Wise Safe Switching (SWSS), has been first established in [4] for the case of single input-single output (SISO) systems, with unknown description. The plant is modeled using a switching multi-linear model, comprising a set of SISO linear systems, which are derived through identification around a sufficiently dense set of nominal operating points. A significant characteristic of SWSS is the application of “common” controllers that achieve the required performance simultaneously for more than one adjacent nominal operating points. Moreover, controller switching is allowed only when the process is close to steady state. This requirement avoids undesirable effects that may come from switching while moving e.g. instability. Thus, the motion between any two different operating points is performed by moving in a step-wise manner between operating areas of an appropriately selected sequence of adjacent nominal operating points. The operating areas are determined with the use of experimental process data. In [4], the performance of SWSS has been studied for the case of the first-order nonlinear process of a CSTR reactor.

In the present work we study the application to a characteristic nonlinear process with second-order dynamics of the SWSS control approach for SISO plants. The process nonlinear description is considered to be unknown for control design, while its behavior is approximated by a set of linear models derived through identification around corresponding operating points. The performance of the SWSS scheme, which is studied using simulation results, seems to be advantageous when compared to a standard switching algorithm, as well as when compared to an approach seeking for “common” (robust) controllers for more than two operating points. The performance of SWSS for plants with higher order dynamics will be studied in future works.

II. STEP-WISE SAFE SWITCHING

In the present section, the SWSS algorithm for SISO systems introduced in [4], is presented in short (see also [7]).

Consider a SISO plant with discrete-time description, where \( y(k) \) denotes the plant’s output and \( u(k) \) denotes the plant’s input. Let \( L = \{ \ell_1, \ell_2, \ldots, \ell_{\mu} \} \) denote a set of nominal operating points of the plant (points of the input-output space where the plant may settle at steady state), where each \( \ell_i, i=1, \ldots, \mu \) is denoted as \( \ell_i = [Y_i, U_i] \), with \( Y_i \) and \( U_i \) denoting the corresponding nominal output and input values. Consider also that the plant’s description is approximated by the following set of linear models, which are determined through identification around the nominal operating points:

\[
S_i : A(q^{-1})\Delta y(k) = B(q^{-1})\Delta u(k - d_i) + C_i(q^{-1})\epsilon_i(k)
\]

where \( \Delta y(k) = y(k) - Y_i \) and \( \Delta u(k) = u(k) - U_i \) denote perturbations of the output and input variables around \( \ell_i \) and \( \epsilon_i(k) \) denotes the unmodeled error or disturbance in \( S_i \). The nonnegative integer \( d_i \) denotes the delay of the model \( S_i \). The operators \( A(q^{-1}), B(q^{-1}) \) and \( C_i(q^{-1}) \) are polynomials of the delay operator \( q^{-1} \) with real coefficients.

For each nominal operating point \( \ell_i \), we determine a pair of operating areas, named target \( (O_i) \) and tolerance \( (\tilde{O}_i) \) operating areas [4], which constitute experimental approximations of the neighborhood of validity of each local linear model \( S_i \). The target operating area is determined as a rectangle in the \((U,Y)\)-space, according to the following rule: For each step transition between an initial operating point \( \rho = [Y_0,U_0] \) within \( O_i \) to a final operating point...
Consider the case where identification is performed around three nominal operating points, deriving the following three corresponding linearized second order discrete-time models:

**Operating Point** \( \ell_1 = [Y, U] = [0.09395, 1.34] \)
\[
\Delta_j y(k) - 1.7619\Delta_j y(k - 1) + 0.7756\Delta_j y(k - 2) = 0.0045\Delta U(k - 1) - 0.0032\Delta U(k - 2) + \epsilon_j(k)
\]
\( S_1 : \]

**Operating Point** \( \ell_2 = [Y, U] = [0.11066, 1.49] \)
\[
\Delta_j y(k) - 1.7870\Delta_j y(k - 1) + 0.7978\Delta_j y(k - 2) = 0.0057\Delta U(k - 1) - 0.0042\Delta U(k - 2) + \epsilon_j(k)
\]
\( S_2 : \]

**Operating Point** \( \ell_3 = [Y, U] = [0.12902, 1.61] \)
\[
\Delta_j y(k) - 1.8073\Delta_j y(k - 1) + 0.8160\Delta_j y(k - 2) = 0.0071\Delta U(k - 1) - 0.0055\Delta U(k - 2) + \epsilon_j(k)
\]
\( S_3 : \]

Note that the sampling period is \( T = 10 \text{[min]} \).

The linear models, derived through the identification procedure, are evaluated experimentally in order to determine the operating areas of the process around which the linear models constitute satisfactory approximations of the nonlinear process. The operating areas of each nominal operating point are determined by comparing the responses of the nonlinear process and the corresponding linear model for step input functions with several amplitudes. According to the above, the target and tolerance operating areas are determined according to the following [4]:

a) **Target operating areas** \( O_i, i = 1, 2, 3 \): Consider that for \( k < 0 \) the process rests at an initial operating point \( \rho_i = [Y, U] \) in \( O_i \). Then, for \( k \geq 0 \) a suitable step input function is applied to the process such that the final operating point is \( \rho_f = [Y, U] \), where \( \rho_f \) lies also within \( O_i \). Let \( V_{ij} = [Y(0) - Y_i, Y(1) - Y_i, ..., Y(N) - Y_i] \) be a vector with elements the deviations of the response \( y(k) \) of the process from the output value \( Y_i \) of the nominal operating point \( \ell_i \), for the sampling instants \( 0, 1, 2, ..., N \), where \( N \) is selected large enough for the process to settle to its steady state value. Consider the linear model \( S_i \), with \( \epsilon_i = 0 \). Apply the

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