Chapter 8

Added Masses of a Propeller

Here we discuss the properties and methods of determination of added masses of a propeller, as well as the influence of the hull on the added masses.

8.1 Forces and Torques of Inertial Nature Acting on a Propeller

Let us introduce the coordinate system \(xyz\) associated to the propeller (Fig. 8.1) such that the origin \(O\) is situated at the axis; the \(Ox\) axis is directed along the propeller axis; the \(Oz\) axis is directed along one of the blades. The number of the blades we denote by \(Z\). If the coordinate system \(xyz\) is rotated around the axis \(Ox\) by the angle \(\beta = 2\pi/Z\), then the axis \(Oz_1\) of the new coordinate system is also directed along a blade of the propeller, i.e., the position of the propeller in the new coordinate system \(xy_1z_1\) is the same as its position under the old coordinate system \(xyz\). Therefore the added masses of the propeller in the new coordinate system \(xy_1z_1\) are the same as the added masses in the coordinate system \(xyz\). Consider kinetic energy of the fluid flow around the propeller (see (1.13)) in the coordinate systems \(xyz\) and \(xy_1z_1\).

Let in coordinate system \(xyz\) the linear and angular velocities of the propeller be denoted by \(u_i\) \((i = 1, \ldots, 6)\). Then in the coordinate system \(xy_1z_1\) these velocities have the form

\[
\begin{align*}
    u'_1 & = u_1; \quad u'_2 = u_2 \cos \beta + u_3 \sin \beta; \\
    u'_3 & = -u_2 \sin \beta + u_3 \cos \beta; \quad u'_4 = u_4; \\
    u'_5 & = u_5 \cos \beta + u_6 \sin \beta; \quad u'_6 = -u_5 \sin \beta + u_6 \cos \beta.
\end{align*}
\]

Kinetic energy of the fluid in the coordinate system \(xyz\) can be written as follows:

\[
2T = \lambda_{11}u_1^2 + \lambda_{22}u_2^2 + \lambda_{33}u_3^2 + 2\lambda_{12}u_1u_2 + 2\lambda_{13}u_1u_3 + 2\lambda_{23}u_2u_3 \\
+ 2u_1(\lambda_{14}u_4 + \lambda_{15}u_5 + \lambda_{16}u_6) + 2u_2(\lambda_{24}u_4 + \lambda_{25}u_5 + \lambda_{26}u_6) \\
+ 2u_3(\lambda_{34}u_4 + \lambda_{35}u_5 + \lambda_{36}u_6) + \lambda_{44}u_4^2 + \lambda_{55}u_5^2 \\
+ \lambda_{66}u_6^2 + 2\lambda_{45}u_4u_5 + 2\lambda_{46}u_4u_6 + 2\lambda_{56}u_5u_6.
\]

Fig. 8.1 Coordinate system associated to a propeller. \( R \) is the maximal radius of the propeller; \( r_0 \) is the radius of the propeller’s hub.

The same kinetic energy in the coordinate system \( xy_1z_1 \) can be written as follows:

\[
2T = \lambda_{11}u_1^2 + \lambda_{22}(u_2 \cos \beta + u_3 \sin \beta)^2 + \lambda_{33}(u_2 \sin \beta + u_3 \cos \beta)^2 + 2\lambda_{12}(u_2 \cos \beta + u_3 \sin \beta)u_1 + 2\lambda_{13}u_1(-u_2 \sin \beta + u_3 \cos \beta) + 2\lambda_{23}(u_2 \cos \beta + u_3 \sin \beta)(-u_2 \sin \beta + u_3 \cos \beta) + 2u_1[\lambda_{14}u_4 + \lambda_{15}(u_5 \cos \beta + u_6 \sin \beta) + \lambda_{16}(-u_5 \sin \beta + u_6 \cos \beta)] + 2(u_2 \cos \beta + u_3 \sin \beta)[\lambda_{24}u_4 + \lambda_{25}(u_5 \cos \beta + u_6 \sin \beta) + \lambda_{26}(-u_5 \sin \beta + u_6 \cos \beta)] + 2(-u_2 \sin \beta + u_3 \cos \beta)[\lambda_{34}u_4 + \lambda_{35}(u_5 \cos \beta + u_6 \sin \beta) + \lambda_{36}(-u_5 \sin \beta + u_6 \cos \beta)] + \lambda_{44}u_4^2 + \lambda_{55}(u_5 \cos \beta + u_6 \sin \beta)^2 + \lambda_{66}(-u_5 \sin \beta + u_6 \cos \beta)^2 + 2\lambda_{45}u_4(u_5 \cos \beta + u_6 \sin \beta) + 2\lambda_{46}u_4(-u_5 \sin \beta + u_6 \cos \beta) + 2\lambda_{56}(u_5 \cos \beta + u_6 \sin \beta)(-u_5 \sin \beta + u_6 \cos \beta).
\]

Expressions (8.1) and (8.2) are identically equal for arbitrary values of \( u_i \) \((i = 1, 2, \ldots, 6)\), since they define the same kinetic energy. Comparing various terms in these formulas we can get relationships between the added masses \( \lambda_{ik} \) \((i, k = 1, 2, \ldots, 6)\), which follow from the discrete rotational symmetry of the propeller.

For two-blade propeller \( Z = 2, \beta = \pi \). The formula (8.2) takes the form

\[
2T = \lambda_{11}u_1^2 + \lambda_{22}u_2^2 + \lambda_{33}u_3^2 - 2\lambda_{12}u_1u_2 - 2\lambda_{13}u_1u_2 + 2\lambda_{23}u_2u_3 + 2u_1(\lambda_{14}u_4 - \lambda_{15}u_5 - \lambda_{16}u_6) - 2u_2(\lambda_{24}u_4 - \lambda_{25}(u_5 \cos \beta + u_6 \sin \beta) - \lambda_{26}u_6) - 2u_3(\lambda_{34}u_4 - \lambda_{35}u_5 - \lambda_{36}u_6) + \lambda_{44}u_4^2 + \lambda_{55}u_5^2 + \lambda_{66}u_6^2 - 2\lambda_{45}u_4u_5 - 2\lambda_{46}u_4u_6 + 2\lambda_{56}u_5u_6.
\]

(8.3)