Today’s engineer is a problem-solver, and the engineering method splits into problem analysis and design followed by solution implementation. More and more the engineer must:

- Decide upon the right questions to be asked.
- Formulate a scientific and engineering description of the questions.
- Use appropriate analysis and software tools to solve the problem.
- Engineer and implement the solution.

The control engineer needs analysis tools to prepare a precise problem formulation and software tools to find a solution to the problem posed. Logical reasoning is needed by the engineer to ensure sound judgement and to work with some sophisticated mathematical methods. In control engineering many of the ideas, concepts and mathematical tools are advanced, but software is available to make the use of these tools easy.

The control engineer must not be fazed by the use of precision mathematics and brilliant software packages. What is important is being able to concentrate on the real issues of the control problem. The toolkit of analysis methods for control engineering is small, but should be thoroughly understood and practised. The software kit used – products like MATLAB (Chapter 3) for example – should be treated like a sophisticated calculator. What is important is the control problem to be formulated and what the solution says about the control issues being studied. In this chapter we describe a small set of tools which form a Control Engineer’s Analysis Toolkit. We can use this chapter either to learn about and practise using the tools or simply to refer to the appropriate section when we need to.

We can even work our way through the whole chapter on the grounds that the key to understanding the theoretical aspects of control engineering and system dynamics is a thorough knowledge of complex numbers and Laplace Transforms. In this case we need to organise our work around the five steps that are needed to acquire a sound knowledge of Laplace transforms for control engineering studies. These steps are shown in Figure 2.1.

The chapter is ordered according to these steps, but we have set it in the form of a set of frequently asked questions on the analysis found in control engineering studies. In this way we can also use the chapter to answer worrying questions.
Learning objectives

- To revise complex number operations.
- To introduce the Laplace transform for signals and systems.
- To introduce an alternative Laplace transform representation for differentiation and integration.
- To examine some formal Laplace transform manipulations.

Step 1

Complex numbers

How many ways of writing complex numbers are there?

There are three representations for complex numbers, these are called Cartesian, polar and complex exponential.

**Cartesian (or rectangular) representation:**

\[ z = a + jb \]

where \( a \) and \( b \) are real numbers

**Polar representation:**

\[ z = r(\cos \theta + j \sin \theta) \]

where \( r \geq 0 \) and \(-\pi < \theta \leq \pi\)

**Complex exponential representation:**

\[ z = re^{j\theta} \]

We should note that we can move from one representation to another and also that we have two graphical interpretations available for use: the Cartesian and the polar versions. We can represent \( z = a + jb \) using the Cartesian plane, as shown in Figure 2.2.