Chapter 2

Network Equilibrium under Cumulative Prospect Theory and Endogenous Stochastic Demand and Supply

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Abstract In this paper we consider a network whose travel demands and road capacities are endogenously considered to be random variables. With stochastic demand and supply the route travel times are also random variables. In this scenario travelers choose their routes under travel time uncertainties. Several evidences suggest that the decision making process under uncertainty is significantly different from that without uncertainty. Therefore, the paper applies the decision framework of cumulative prospect theory (CPT) to capture this difference. We first formulate a stochastic network model whose travel demands and link capacities follow lognormal distributions. The stochastic travel times can then be derived under a given route choice modeling framework. For the route choice, we consider a modeling framework where the perceived value and perceived probabilities of travel time outcomes are obtained via transformations following CPT. We then formulate an equilibrium condition similar to that of User Equilibrium in which travelers choose the routes that maximizes their perceived utility values in the face of transformed stochastic travel times. Conditions are established guaranteeing existence (but not uniqueness) of this equilibrium. The paper then proposes a solution algorithm for the proposed model which is then tested with a test network.

1. Introduction

Travelers routinely experience variability in network travel times, and although they may not be able to determine the cause of this variability, they can account for it in their travel choices. Variations in travel time can arise from an array of different sources: the volume of network traffic is generated by individuals’ activity schedules, resulting in traffic flows that vary from day to day; the capacity of the road network is degraded unpredictably by accidents and incidents.

Several researches have attempted to include travel time variability and uncertainty into travel choice decision models (route, mode, or departure time) (Noland...
These analyses introduced different approaches to capture how travelers compare distributions of travel times on alternative routes within their decision making process. A common formulation is to represent the travel time distribution by the mean travel time plus an additional component related to the variance of travel times (Noland and Small 1995), to the probability of being late (Watling 2006), or to the safety margin required to arrive on time (Lam et al. 2008; Lo et al. 2006). Alternatively, a cost can be associated with the travel time uncertainty according to travelers’ risk attitudes; assuming that travelers accurately perceive the travel time distribution (Szeto et al. 2006) or play out all possible scenarios before making a choice (Bell and Cassir 2002). In all cases, travelers’ decisions are based on the actual travel time distribution and alternatives chosen to maximize expected utility. An alternative perspective is that travelers do not accurately perceive the travel time distribution. Mirchandani and Soroush (1987) assume that different travelers each perceive the mean and variance of travel time differently, giving rise to a distribution of perceived travel time means and variances. On this basis they compute an equilibrium assignment.

In the area of choice modeling, cumulative prospect theory (CPT) sets out a framework for decision making based on transformations of both the distribution of outcomes’ utilities and probabilities (Tversky and Kahneman 1992). CPT as a choice model has been applied in the transport field (for example, see Avineri and Prashker 2003; Avineri 2006; Michea and Polak 2006). In particular Connors and Sumalee (2009) proposed a general approach to represent travelers’ route choice in the face of stochastic travel times; not only allowing for subjective perceived costs associated with the distribution of travel times, but also allowing for subjective perception of their uncertainty (similar to CPT). They propose a general equilibrium condition derived for a network whose travel times arise from arbitrary probability distributions, with travelers’ choosing their routes based on subjective perceptions of the route travel time distributions. This formulation accommodates monotonic nonlinear transformations for the (i) perceived disutilities in terms of actual travel times and (ii) perceived risk in terms of the actual travel time probabilities. However, their model assumes exogenous source of uncertainty imposed upon the route travel times.

In this paper, we extend their analysis to the case where the stochastic route travel times are caused by the endogenous random travel demand and link capacity. We assume that the origin-destination (OD) travel demands and link capacities follow lognormal distributions. The paper is structured as follows. Section 2 introduces notation necessary for the formulation of network equilibrium and the two transformations of the travel time distributions. In addition, the section describes the assumption of lognormal random demand and the derivation of the lognormal route travel time. Motivated by CPT, the perceived value of a route travel time (following lognormal distribution) is defined in Section 3. Section 4 then presents the equilibrium formulation and analyzes existence. Numerical examples are presented in Section 5 before concluding remarks in Section 6.