Transform coding (TC) is a method that transforms a source signal into another one with a more compact representation. The goal is to quantize the transformed signal in such a way that the quantization error in the reconstructed signal is smaller than directly quantizing the source signal.

5.1 Transform Coder

Transform coding is block-based, so the source signal \( x(n) \) is first grouped into blocks, each of which consists of \( M \) samples, and is represented by a vector:

\[
x(n) = [x_0(n), x_1(n), \ldots, x_{M-1}(n)]^T \\
= [x(nM), x(nM - 1), \ldots, x(nM - M + 1)]^T.
\]  

The dimension \( M \) is called block size or block length. For a linear transform, the transformed block is obtained as

\[
y(n) =Tx(n),
\]

where

\[
y(n) = [y_0(n), y_1(n), \ldots, y_{M-1}(n)]^T
\]

is called the transform of \( x(n) \) or transform coefficients and the \( M \times M \) matrix

\[
T = \begin{bmatrix}
t_{0,0} & t_{0,1} & \cdots & t_{0,M-1} \\
t_{1,0} & t_{1,1} & \cdots & t_{1,M-1} \\
\vdots & \vdots & \ddots & \vdots \\
t_{M-1,0} & t_{M-1,1} & \cdots & t_{M-1,M-1}
\end{bmatrix}
\]

is called the transformation matrix or simply the transform. This transform operation is shown in the left of Fig. 5.1.
Transform coding is shown in Fig. 5.1. The transform coefficients \( y(n) \) are quantized into quantized coefficients \( \hat{y}(n) \) and the resulting quantization indexes transmitted to the decoder. The decoder reconstructs from the received indexes the quantized coefficients \( \hat{y}(n) \) through inverse quantization. This process may be viewed as if the quantized coefficients \( \hat{y}(n) \) are received directly by the decoder.

The decoder then reconstructs an estimate \( \hat{x}(n) \) of the source signal vector \( x(n) \) from the quantized coefficients \( \hat{y}(n) \) through an inverse transform

\[
\hat{x}(n) = T^{-1} \hat{y}(n),
\]

where \( T^{-1} \) represents the inverse transform. The reconstructed vector \( \hat{x}(n) \) can then be unblocked to rebuild an estimate \( \hat{x}(n) \) of the original source signal \( x(n) \).

A basic requirement for a transform in the context of transform coding is that it must be invertible

\[
T^{-1}T = I,
\]

so that an source block can be recovered from its transform coefficients in the absence of quantization:

\[
x(n) = T^{-1}y(n).
\]

**Orthogonal transforms** are most frequently used in practical applications. To be orthogonal, a transform must satisfy

\[
T^{-1} = T^T,
\]

or

\[
T^T T = I.
\]