Chapter 14
Generalized Beams and Continua. Dynamics of Reticulated Structures

Claude Boutin, Stéphane Hans, and Céline Chesnais

Abstract This paper deals with the dynamic behavior of periodic reticulated beams and materials. Through the homogenization method of periodic discrete media the macro-behavior is derived at the leading order. With a systematic use of scaling, the analysis is performed on the archetypical case of symmetric unbraced framed cells. Such cells can present a high contrast between shear and compression deformability, conversely to “massive” media. This opens the possibility of enriched local kinematics involving phenomena of global rotation, inner deformation or inner resonance, according to studied configuration and frequency range.

14.1 Introduction

This paper deals with the macroscopic dynamic behavior of periodic reticulated structures and materials widely encountered in mechanical engineering. Periodic lattices have been studied through various approaches such as transfer matrix, variational approach [8], finite difference operator, cf. [10]. Asymptotic methods of homogenization [11] initially developed for periodic media, were extended to multiple parameters and scale changes by [5] and adapted to periodic discrete structures by [3], then [9]. Those studies aim at relate the local structure and the global behavior.

The structural morphology of reticulated media makes that the basic cells can present a high contrast between shear and compression deformability (conversely to

C. Boutin · S. Hans · C. Chesnais
DGCB, FRE CNRS 3237, Ecole Nationale des Travaux Publics de l’Etat, Université de Lyon, Lyon, France
e-mail: claude.boutin@entpe.fr
S. Hans
e-mail: stephane.hans@entpe.fr
C. Chesnais
e-mail: celine.chesnais@entpe.fr

“massive” media). This opens the possibility of enriched local kinematics involving phenomena of global rotation, inner deformation or inner resonance, according to studied configuration and frequency range. A numerical illustration of these atypical situations is given in Fig. 14.1 that shows the some unusual macroscopic modes. The present study investigates and summarizes those phenomena by a systematic analysis performed on the archetypical case of symmetric unbraced framed cells [1, 7]. Assuming the cell size is small compared to the wavelength, the homogenization of periodic discrete media leads to the macro-behavior at the leading order. The paper is organized as follows. Section 14.2 gives an overview of the method and the assumptions. In Sect. 14.3, several beam modeling under transverse vibrations are established by varying the properties of the basic frame elements, and the frequency range. Section 14.4 is devoted to longitudinal vibrations. Section 14.5 focuses on the analogy between these results and the mechanics of generalized continua.

14.2 Overview of Discrete Homogenization

The analysis of periodic beam lattices is performed in two steps [12]: first, the discretization of the balance of the structure under harmonic vibrations; second, the homogenization, leading to a continuous model elaborated from the discrete description. An outline of this method is given hereafter.

Discretization of the Dynamic Balance Studied structures (Fig. 14.2) are made of plates behaving as Euler–Bernoulli beam in out-of-plane motion, and assembled with rigid connections. The motions of each extremity connected to the same node are identical and define the discrete nodal kinematic variables of the system. The discretization consists in integrating the dynamic balance (in harmonic regime) of the beams, the unknown displacements and rotations at their extremities taken as boundary conditions. Forces applied by an element on its extremities are then ex-