Chapter 7
Equilibrium Problems with Equilibrium Constraints

7.1 Introduction

The previous chapter presented mathematical programs for solving leader-follower (Stackelberg) games when a single leader correctly anticipates the equilibrium reaction of followers, who in turn naively believe that the leader’s decisions are exogenous and fixed. This chapter introduces a type of mathematical program that is useful for modeling such games when there is more than one leader, and one wants to find an equilibrium among them: Equilibrium Problems with Equilibrium Constraints (EPECs). First, we present a general EPEC formulation (Section 7.2) and the basic diagonalization approach to solving EPECs, including a simple example. We then summarize some of their many applications to energy markets (Section 7.3), including the three examples of energy market EPECs that we feature in this chapter.

The detailed examples of Sections 7.4-7.6 include both formulations and simple numerical examples that the reader can replicate. These EPECs represent two types of hierarchical (leader-follower) relationships in the energy supply chain: between oligopolistic power generators and a transmission system operator (Sections 7.4 and 7.5) and among oligopolistic producers who play successive capacity expansion—spot market games (Section 7.6). The models in Sections 7.4 and 7.5 represent short-run market operations situations in which generators submit MW quantity schedules and price offers, respectively, to the grid operator, who then operates the system optimally subject to those schedules or offers. In these multiple leader-follower games, the generators anticipate correctly how the grid operator’s calculation of market prices will react to those schedules and offers. However, the generator leaders play a Nash game among themselves.

In the long-run investment game of Section 7.6, we disregard the operation of the transmission grid and instead consider a situation in which generators can invest in capacity and anticipate correctly how those investments affect the short-run operations of the market. However, the generators are Nash
players with respect to each other’s capacity. The interaction of the multiple investors (leaders) with the short-run operations (followers) yields an EPEC model structure.

Diagonalization is used to solve all the examples in the chapter, in the case where pure strategy solutions exist; another, more advanced algorithm that can be used to solve EPECs is presented in Section 9.4.2. Where pure strategy solutions do not exist in our examples, we use the bimatrix approach of Section 4.2.4 to obtain mixed strategy equilibria.

7.2 The EPEC Problem

7.2.1 Problem Statement and Diagonalization Algorithm

Chapter 6 introduced several closely related mathematical programming problems that involve optimization of an objective (upper-level problem) subject to the solution of another problem (lower-level problem). If the lower-level is an optimization problem, then the overall problem is an OPcOP in that chapter’s terminology. Alternatively, the lower-level might instead be a complementarity problem that represents an equilibrium among the followers and which might not necessarily be expressible as the solution of an optimization problem, in which case the overall two-level problem is referred to as an MPEC in Chapter 6. If the lower-level problem is an optimization problem, it is usually represented by its first-order conditions (in the form of a complementarity problem). Chapter 6 calls the overall problem an MPCC. Less commonly, the solution might be represented as three sets of constraints: one for primal variables of the subproblem, one for its dual variables, and finally a zero duality gap condition that sets the primal and dual objectives equal (termed a MPPDC in Chapter 6). If the first-order conditions or primal-dual conditions of an OPcOP lower-level optimization problem are viewed as equilibrium conditions for that single optimization, then the OPcOP is itself an MPEC. Therefore, for the purposes of this chapter, we will refer to all of those problems as MPECs, even though, strictly speaking, some readers might prefer to use the term MPEC for only a subset of the two-level problems of Chapter 6.

This chapter is concerned with the formulation and calculation of an equilibrium among several MPECs that share the same lower-level problem, whether an optimization or an equilibrium problem. In game theoretic terms, the problem is to obtain a Nash equilibrium among multiple leaders of a Stackelberg leader-follower problem, where there is a shared single follower or set of followers. A Stackelberg game represents a situation of asymmetric information in which the leader(s) correctly anticipate the reaction of