Chapter 9
Some Advanced Algorithms for VI Decomposition, MPCCs and EPECs

9.1 Introduction

In this chapter, we present several advanced algorithms that can be useful for the solution of some of the models discussed in this book.

Section 9.2 discusses several decomposition algorithms for variational inequality (VI) models: an extension of the classical Dantzig-Wolfe decomposition algorithm \[6\] from the realm of optimization models to VIs; simplicial decomposition of VIs, which relies on the column generation idea of Dantzig-Wolfe decomposition; and an extension of Benders decomposition from optimization models to VIs, explained as an application of Dantzig-Wolfe decomposition to the dual VI of a given primal VI model. The cobweb decomposition algorithm is also briefly described.

Section 9.3 presents several methods to solve mathematical programs with complementarity constraints (MPCCs): regularization, which solves a sequence of NLPs with relaxed complementarity constraints that are tightened at each step of the sequence; penalization, which solves a sequence of NLPs that have no complementarity constraints but instead penalize violations of complementarity, in the objective function; and sequential quadratic programming, which solves a sequence of quadratic programs that approximate the given MPCC. Several other methods to solve MPCCs are briefly described.

Section 9.4 discusses two methods to solve equilibrium programs with equilibrium constraints (EPECs): a brief discussion of diagonalization, which is described more fully in Chapter 7; and the reformulation of an EPEC as an NLP.
9.2 Decomposition Algorithms for VIs

Sometimes, a VI model has a structure that is close to the structure of a VI that would be easier to solve. For example, it can happen that if a few complicating constraints were not present in the model, then the model would split apart into several smaller, independent subproblems that could be solved more easily than the original large model, and solution of the separate subproblems could perhaps be done simultaneously, in parallel on separate computers. Other VI models may have complicating variables, i.e., if these variables could be fixed at known values, then the model would be easier to solve, perhaps because it breaks apart into separate smaller subproblems. Decomposition algorithms have been developed for optimization problems – e.g., the famous method of Dantzig and Wolfe [6] for LPs – in order to take advantage of such structures, and some of these algorithms have been extended to the realm of VI models. See [5] for a thorough description of many decomposition algorithms as applied to optimization models.

There is sometimes another motivation to use decomposition techniques, even if decomposition does not speed up the solution process. Murphy and Mudrageda [19] explain that splitting the very large energy market model of the U.S. Department of Energy into separate submodels for different fuels (e.g., electricity, natural gas, etc.) allows different teams of experts to build and maintain the submodels; decomposition techniques are used to bring the separate pieces together into a consistent multi-fuel equilibrium.

In this section, we present three decomposition techniques for VIs – Dantzig-Wolfe, simplicial, and Benders decomposition – and we give an overview of cobweb decomposition. We show the details of Dantzig-Wolfe decomposition for VIs, and we explain simplicial and Benders techniques by deriving them based on Dantzig-Wolfe decomposition.

As an introduction and review of the basic ideas of Dantzig-Wolfe decomposition for optimization models, we begin with a simple illustrative example of Dantzig-Wolfe decomposition applied to an LP. Following the LP example, we present an illustrative example of a stochastic power VI model which is used to illustrate Dantzig-Wolfe, simplicial, and Benders decomposition in later subsections. The stochastic power model is a simplified version of a VI model from Chapters 4 and 5, and in [11].

9.2.1 Illustrative Example. Dantzig-Wolfe Decomposition of a Simple LP

Consider the following LP in four variables.

Minimize $4x_1 + x_2 + 2x_3 + 6x_4$  \hspace{1cm} (9.1a)