Chapter 1
On Practical Product Form Characterizations

Nico M. van Dijk

Abstract
Do we have a product form?
If so, how is it characterized?
If not, how can product forms still be useful?
The first question is not be that easy as it seems. The answer might depend on the state level of interest, the service assumptions imposed and the system restrictions or flexibilities in order. This chapter aims to address these question in two parts:

A: Product Forms: A Single Station
B: Product Forms: Tandem and Cluster Structures

In A just a single service station is studied to show how different levels of a state description and notions of balance may lead to analytic forms that can be referred to as 'product forms'. It covers simple birth-death type systems, forms of access blocking for multi-class stations, and symmetric up to so-called invariant disciplines.

In B just a tandem type structure (that is with consecutive service stations) and some Jacksonian cluster extensions are dealt with to show:

(i) The effect on the existence of a product form under practical phenomena as blocking and service sharing
(ii) How this existence can be characterized
    • in an analytic manner by 'adjoint' reversibility
    • by simple physical station or cluster 'outrate=inrate' principles
(iii) The practical way in which these insights can be used to obtain simple product form bounds for unsolvable systems

Nico M. van Dijk
University of Amsterdam, Roetersstraat 11, 1018 WB Amsterdam, The Netherlands
e-mail: n.m.vandijk@uva.nl

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A: Product Forms: Single Station Hierarchy

1.1 Introduction

No doubt that the popularity of queueing networks, next to its potential for modeling a variety of practical service networks, is largely due to the existence of the well-known product form expressions ever since the pioneering work by Jackson (1957). In several subsequent chapters, various generalizations of this product form in more abstract settings will be provided (and different lights will be shed on its validity). For further interest in these generalizations the reader is referred to these chapters. Nevertheless, as of today, a number of questions related to the existence of product forms still seem to be open, most notably among which:

- The simple question whether a specific network of interest has a product form or not.
- What is actually meant by a product form, is it uniquely defined, and under which conditions and to what extent or level does it apply.
- Last but not least, how can we guess and verify a particular product form in a down-to-earth manner.

This chapter merely aims to provide some more insights and partial answers for these questions. It also aims to do so in an instructive manner by following the down-to-earth approach of straightforward verification of balance equations. As such, it will be far from exhaustive. Roughly the objectives are:

Objectives.

1. To show the verification and the relation of product forms with different (levels of) partial balance and to emphasize the physical interpretation of these partial balances.
2. To show (a hierarchy of) different levels of product forms and partial balance as depending on state description and conditions satisfied.
3. To show the characterization of these partial balances and its related product forms by means of reversibility and, as will be called: 'adjoint reversibility'.
4. To provide instructive as well as 'non-standard' product form examples, some of which might still be regarded as 'new’, or at least which have not been reported explicitly.
5. To illustrate the application of these product form insights such as to provide simple and possibly insensitive product form bounds for practical non-product form systems.