Chapter 3
Uncertainty Characterization via Scenarios

3.1 Introduction

Many engineering and science problems are subject to uncertainty due to the inherent randomness of natural phenomena and/or to the imperfect knowledge of the variables determining the functional state of the human-created structures. In this context, computational methods that tackle uncertainty allow engineers and scientists to propose solutions less sensitive to environmental influences, while achieving simultaneously cost reduction, profit gains, and/or reliability improvement.

Decision-making problems related to electricity markets are not exempt from uncertainty. The own rules governing the functioning of these markets can be deemed responsible for the existence of uncertainties conditioning market agents’ behavior. For instance, energy prices are known after producers and consumers submit their selling offers and purchasing bids, respectively, to the electricity market. As a result, decisions on the amount and price of the energy to be sold or purchased are irremediably made with inaccurate knowledge of the final market outcome. Likewise, the time gap existing between agreements on energy transactions and their physical implementation causes that a producer must face the trading process with a certain degree of uncertainty about the availability of its power sources.

In Chapter 2, stochastic programming was introduced as an efficient tool to optimize under uncertainty, that is, to find optimal decisions in problems involving uncertain data. Within a modeling framework based on stochastic programming, input data affected by uncertainty are conceptually described as stochastic processes. A stochastic process $\lambda$ is defined as a collection of dependent random variables $\lambda = \{\lambda_t, t \in T\}$. That is, for each $t$ in the index set $T$, $\lambda_t$ is a random variable. We often interpret $t$ as time and call $\lambda_t$ the state of the process at time $t$. A stochastic process $\lambda$ is said to be continuous or discrete depending on whether its component random variables $\{\lambda_t, t \in T\}$ are continuous or discrete, respectively.

We present below examples of both a continuous and a discrete stochastic process.

**Illustrative Example 3.1 (A continuous stochastic process).** If we denote the wind speed in hour $t$ at a given site $A$ by $v^A_t$, then the set of random variables $V^A = \{v^A_t, t = 1, 2, \ldots, 24\}$ describing wind speed at site $A$ throughout a day is a continuous stochastic process.

**Illustrative Example 3.2 (A discrete stochastic process).** Consider the binary variable $u_{it}$, which is equal to 1 if generating unit $i$ is available in hour $t$ and 0 otherwise. The collection of random variables $U_i = \{u_{it}, t = 1, 2, \ldots, 168\}$ representing the availability of unit $i$ throughout a week is a discrete stochastic process.

Note that a discrete stochastic process can be represented by a finite set of actual vectors, referred to as scenarios, resulting from the combinations of all the discrete values that its component random variables can adopt. In mathematical terms, if $\lambda$ is a discrete stochastic process, it can be expressed as $\lambda = \{\lambda(\omega), \omega = 1, 2, \ldots, N_\Omega\}$, where $\omega$ is the scenario index and $N_\Omega$ is the number of possible scenarios. In order for the discrete stochastic process to be perfectly determined, a probability of occurrence $\pi(\omega)$ needs to be associated with each realization $\lambda(\omega)$ such that $\sum_{\omega=1}^{N_\Omega} \pi(\omega) = 1$.

The following example illustrates the mathematical representation of a discrete stochastic process.

**Illustrative Example 3.3 (Mathematical representation of a discrete stochastic process).** Let us consider the discrete stochastic process $U_i$ representing the availability of generating unit $i$ throughout a time horizon comprising two hourly periods. Process $U_i$ can be mathematically expressed as $U_i = \{U_i(\omega) = [u_{i1}(\omega), u_{i2}(\omega)], \omega = 1, 2, 3, 4\}$, where binary variable $u_{it}(\omega)$, $t = 1, 2$, is equal to 1 if unit $i$ is available in time period $t$ and scenario $\omega$, and 0 otherwise. Specifically, each scenario $U_i(\omega)$ can be written in the form

$U_i(1) = [1, 1]$, with a probability of occurrence $\pi(1) = 0.5$

$U_i(2) = [0, 1]$, with $\pi(2) = 0.2$

$U_i(3) = [1, 0]$, with $\pi(3) = 0.2$

$U_i(4) = [0, 0]$, with $\pi(4) = 0.1$.

Note that the above probabilities of occurrence are merely illustrative. In practice, these probabilities are obtained through a rigorous analysis that accounts for the failure ratio of the generating unit under consideration. Further details on this issue can be found in Subsection 3.2.3.

Solving optimization problems where uncertainty on input data is modeled by continuous stochastic processes is very difficult, or even impossible in