Chapter 4

An Aperitif on Arbitrage

As a simple introduction to the concept of arbitrage and how to use the assumption of no arbitrage to price assets, let us briefly discuss the pricing of forward contracts and some simple issues related to option pricing.

4.1 Background on Forward Contracts

We begin with the necessary background on forward contracts.

Forward Contracts

A forward contract is an agreement to buy a certain quantity of an asset, called the underlying asset at a given price \( K \), called the settlement price or delivery price to be paid at a given time \( T \) in the future, called the settlement date or delivery date. Entering a forward contract does not require any initial purchase price—it is free.

The party that agrees to buy the asset is taking the long position on the contract and is said to be the buyer of the contract. The party that agrees to sell the asset is taking the short position on the contract and is said to be the seller of the contract.

Futures Contracts

In contrast to plain-vanilla forward contracts as described above, a futures contract is a forward contract with a number of constraints and a much more complicated payoff model. Indeed, futures contracts seldom come to maturity, that is, very few (perhaps on the order of 1 or 2 percent) of all futures contracts survive to the delivery date. The main properties of futures contracts are as follows.

1) Futures contracts trade on an organized exchange. For example, the Chicago Board of Trade (CBT or CBOT) is the largest futures exchange.

2) Futures contracts have standardized terms, specifying the amount and precise type of the underlying, the delivery date and delivery price. Just like you can only buy bolts of specific lengths and diameters at the hardware store, you can only buy futures contracts with specific terms.
3) Performance (delivery of losses or gains) of futures contracts is guaranteed by a clearinghouse.

4) The purchase of a futures contract requires that the buyer post margin, that is, some amount of money to cover potential day-to-day price changes.

5) Futures markets are regulated by a government agency, whereas forward contracts are largely unregulated.

6) Futures contracts can be closed (terminated) either by delivery, by offset (that is, by a reversing trade that cancels both contracts) or by exchange-for-physical (which is a form of “settle up early” arrangement).

We will not discuss the details of futures contracts in this book.

**Forward Prices**

Consider forward contracts for a given underlying (such as wheat) that have a given delivery date $T$ (such as December 2003). At any time $t < T$, one can potentially enter into such a contract. Of course, the delivery price will depend on the time $t$ of formation of the contract, so we will denote it by $F_{t,T}$. This would-be delivery price is called the forward price of the contract.

For example, on July 1 the forward price of a contract to deliver 5000 bushels of wheat in September might be 170 cents (per bushel). A week later, the forward price for such a contract might be 168 cents.

**Spot Prices**

In contrast to forward prices, the spot price $S_t$ of an asset at a given time $t$ is the price of the asset at that time for immediate delivery. For example, we can speak of the current spot price of a bushel of wheat. We can also speak of the spot price of wheat in one month. This is the price that investors would pay in one month for immediate delivery at that time. Of course, at the present time, this spot price is unknown.

**4.2 The Pricing of Forward Contracts**

To determine the forward price of a forward contract, we can use a simple no-arbitrage argument. Suppose that the forward contract is for one “share” of an asset whose initial price is $S_0$. (One share of a wheat contract is 5000 bushels of wheat, for example.) Consider the following two portfolios.