Effect of functionally graded materials on resonances of rotating beams

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NOMENCLATURE

\( A \), area of the beam cross section  
\( a, C, m, n \), constant numerical parameters  
\( b \), hub radius  
\( E \), Young’s modulus (\( E_a, E_b, E_r \) - distinct values of Young’s modulus for different FGM cases)  
\( F(t) \), system generalized external forcing  
\( I \), area moment of inertia of the beam cross section  
\( J \), mass moment of inertia of the beam and hub about \( \overline{p}_3 \)  
\( K \), system generalized stiffness  
\( L \), beam length  
\( L_\sigma \), Lagrangian  
\( M \), system generalized mass  
\( r \), distance from hub of a generic point \( P \) on the beam (undeformed configuration)  
\( R_\sigma \), beam cross section radius  
\( T \), kinetic energy  
\( t \), time  
\( V \), potential energy  
\( V_a \), potential energy due to axial elongation  
\( V_r \), potential energy due to bending  
\( x \), longitudinal coordinate (along \( \overline{a}_1 \) )  
\( w \), beam deflection in the \( \overline{a}_2 \) direction  
\( \overline{a}_1, \overline{a}_2, \overline{a}_3 \), reference system attached to undeformed configuration of the beam  
\( \overline{A}_p \), acceleration of a generic point \( P \) on the beam  
\( \overline{n}_1, \overline{n}_2, \overline{n}_3 \), inertial reference system  
\( \overline{R}_p \), position of a generic point \( P \) on the beam  
\( \overline{V}_p \), velocity of a generic point \( P \) on the beam  
\( \beta, \lambda, \sigma_1 \), constant numerical parameters  
\( \eta \), function of time  
\( \theta \), hub angular position  
\( \rho \), mass density (\( \rho_0, \rho_b, \rho_r \) - distinct values of mass density for different FGM cases)
Radially rotating beams attached to a rigid stem occur in several important engineering applications, such as helicopter and turbine blades and certain aerospace applications. In most studies the beams have been treated as homogeneous. Here, with a goal of system improvement, non-homogeneous beams made of functionally graded materials are explored. Effects on natural frequency and coupling between rigid and elastic motions are investigated. Euler-Bernoulli theory, with Young’s modulus and density varying in a power law fashion, together with an axial stiffening effect, are employed. The equations of motion are derived using a variational method and an assumed mode approach. Results for the homogeneous and non-homogeneous cases are treated and compared. Preliminary results show that allowing the Young’s modulus and the density to vary by approximately 2.15 and 1.15 times, respectively, gives an increase of 28% in the lowest bending natural frequency of the beam, an encouraging trend.

INTRODUCTION

Rotating machinery form an important part of engineering and radially rotating beams constitute a major category of such systems. For instance, rotor blades, propellers and turbines fall into this category. For vibration control, it is important to identify possible system resonances and, if required and possible, change these values.

Extensive work on these types of problems has been done in the aerospace literature. Comprehensive reviews can be found in the papers of Kane and Ryan [1] and Haering et al. [2]. They, and others, showed that at high speeds the rotating structure can be prone to instabilities. It is assumed here that the rotational speeds are small enough that no instabilities are encountered.

There are numerous works on vibrations of radially rotating beams (uniform beams, beams including pre-twisted and tapered beams). Two classes of problems arise, namely, prescribed motions and prescribed torques. Earlier studies on the former type of problem can be found in the texts by Putter and Manor [3], Hoa [4], Hodges and Rutkowski [5] and Hodges [6]. Putter and Manor used a finite element approach to obtain the natural frequencies and mode shapes of the beam, including shearing forces, rotary inertia and varying centrifugal forces. Hoa also utilized a finite element approach for the same objective, but effects of root radius, setting angle and tip mass were included. Hodges used asymptotic expansions to obtain an approximate value for the fundamental frequency of a uniform beam and Hodges and Rutkowski used a finite element approach to calculate the eigenvalues and eigenvectors of the beam including different hub radii, tapered beams and beams with discontinuities. Kojima [7] investigated the transient flexural vibrations of a beam / mass system attached to a rotating rigid body.

The prescribed torque problem has been studied by, for example, Yigit et al. [8], a work which the current closely follows. In that work the flexural motion of a rotating beam was investigated by using a specified torque profile to drive the rotating body (so that the rigid body motion was not known a priori).

Lee et al. [9] presented experimental results confirming that centrifugal effects cannot be neglected, even at first order, when modeling these systems.

Models utilizing a Timoshenko beam type approach (other than Euler-Bernoulli) are also numerous. See, for example, the work of Lin and Hsiao [10] which investigates the effect of Coriolis force on the natural frequencies of the rotating beam.

More involved models including base excitation can be found in references [11] and [12].