CHAPTER 5
WAVELET BASED NONLINEAR IDENTIFICATION

5.1 Introduction

The approximation of general continuous functions by nonlinear networks has been widely applied to system modelling and identification. Such approximation methods are particularly useful in the black-box identification of nonlinear systems where very little a priori knowledge is available. For example, neural networks have been established as a general approximation tool for fitting nonlinear models from input output data on the basis of the universal approximation property of such networks. There has also been considerable recent interest in identification of general nonlinear systems based on radial basis networks (Poggio and Girosi, 1990a,b), fuzzy sets and rules (Zadeh, 1994), neural-fuzzy networks (Brown and Harris, 1994; Wang et al., 1995) and hinging hyperplanes (Breiman, 1993).

The recently introduced wavelet decomposition (Grossmann and Morlet, 1984; Daubechies, 1988; Mallat, 1989a; Chui, 1992; Meyer, 1993; IEEE, 1996) also emerges as a new powerful tool for approximation. In recent years, wavelets have become a very active subject in many scientific and engineering research areas. Wavelet decompositions provide a useful basis for localised approximation of functions with any degree of regularity at different scales and with a desired accuracy. Recent advances have also shown the existence of orthonormal wavelet bases, from which follows the variability of rates of convergence for approximation by wavelet based networks. Wavelets can therefore be viewed as a new basis for representing functions. Wavelet based networks (or simply wavelet networks) are inspired by both feedforward neural networks and wavelet decompositions. They have been introduced for the identification of nonlinear static systems (Zhang and Benveniste, 1992) and nonlinear dynamical systems (Coca and Billings, 1997; Liu et al., 1998, 2000).

This chapter presents a wavelet network based identification scheme for nonlinear dynamical systems. Two kinds of wavelet networks are studied: fixed and variable wavelet networks. The former are used for the case where the estimation accuracy is assumed to be achieved by a known resolution scale. But, in practice, this assumption is not realistic because the nonlinear function to be identified is unknown and the system operating point may change with time. Thus, variable wavelet networks are introduced to deal with this problem. The basic principle of the variable wavelet network is that the number of wavelets in the network can either be increased or decreased over time according to a
design strategy in an attempt to avoid overfitting or underfitting. In order to model unknown nonlinearities, the variable wavelet network starts with a lower resolution scale and then increases or reduces this according to the novelty of the observation, which is ideally suited to on-line identification problems. The objective of variable wavelet networks is to gradually approach the appropriate network complexity that is sufficient to provide an approximation to the system nonlinearities.

The parameters of the wavelet network are adjusted by adaptation laws developed using a Lyapunov synthesis approach. The identification algorithm is performed over the network parameters by taking advantage of the decomposition and reconstruction algorithms of a multiresolution decomposition when the resolution scale changes in the variable wavelet network. Combining the wavelet network and Lyapunov synthesis techniques, the identification algorithm developed for continuous dynamical nonlinear systems guarantees the stability of the whole identification scheme and the convergence of both the parameters and estimation errors. The wavelet network based identification scheme is realised using B-spline wavelets and it is shown how to calculate decomposition and reconstruction sequences needed for identification using variable wavelet networks.

5.2 Wavelet Networks

Wavelets are a class of functions that have some interesting and special properties. Some basic concepts about orthonormal wavelet bases will be introduced initially. Then the wavelet series representation of one-dimensional and multidimensional functions will be considered. Finally, wavelet networks are introduced.

5.2.1 One-dimensional Wavelets

The original objective of the wavelet theory is to construct orthogonal bases in $L_2(\mathbb{R})$. These bases are constituted by translations and dilations of the same function $\psi$. It is preferable to take $\psi$ as localised and regular. The principle of wavelet construction is the following:

(a) the function $\phi(x - k)$ are mutually orthogonal for $k$ ranging over $\mathcal{N}$;
(b) $\phi$ is a scaling function and the family $\phi(2^j x - k)$ constitutes an orthogonal basis of $L_2(\mathbb{R})$;
(c) the wavelet is defined as $\psi$ and the family $\psi(2^j x - k)$ constitutes an orthogonal basis of $L_2(\mathbb{R})$.

It can also be proved that the family $\{\phi(2^{j_0} x - k), \psi(2^j x - k), \text{ for } j \geq j_0\}$ also forms an orthogonal basis of $L_2(\mathbb{R})$.

The wavelet subspaces $W_j$ are defined as

$$W_j = \{\psi(2^j x - k), \quad k \in \mathcal{N}\} \quad (5.1)$$