Chapter 2

State representation of continuous and discrete systems

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The state representation uses the matrix algebra for the systems representation. The representation of monovariable systems easily extends, by this method, to multivariable systems.

1. State representation of continuous systems

1.1. Heuristic approach

Let us consider the example of a direct current motor, the transfer function of which is established below.

\[ E(p) = U(p) - r I(p) - L p I(p) \]

\( J \) : Rotor inertia moment,
\( r, L \) : Armature resistance and inductance,
\( f \) : Hydraulic friction coefficient of the rotor,
\( k \) : Couple coefficient.
\[ E(p) = k \Omega(p) \]
\[ Cm(p) = k I(p) = J p \Omega(p) + F \Omega(p) \]
i.e., after some calculations:
\[ M(p) = \frac{\Omega(p)}{U(p)} = \frac{k}{k^2 + (r + L p)(J p + F)} \]
i.e.,
\[ M(p) = \frac{b_0}{p^2 + a_1 p + a_0} \]
where
\[
\begin{align*}
  b_0 &= \frac{k}{L J} \\
  a_0 &= \frac{k^2 + r F}{L J} \\
  a_1 &= \frac{F}{J} + \frac{r}{L}
\end{align*}
\]
Let us note:
\[
  Y(p) = \Omega(p) : \text{process output}, \\
  U(p) : \text{process input}.
\]
\[ b_0 U(p) = (p^2 + a_1 p + a_0) Y(p) \]
Which leads to the following differential equation, provided that the initial conditions are null.
\[
\frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_0 u(t)
\]
The system is order 2, requiring 2 state variables, \( x_1 \) and \( x_2 \).

The state variables represent the integrators' outputs.
\[ x_1(t) = y(t) ; \quad x_2(t) = \dot{x}_1(t) \]