12.1 Passivity and Optimal Control of Singular Biological Complex Systems

Passivity and passive control research for singular systems has been effectively developed in [22, 6, 9, 20, 2, 14, 13, 3, 18, 19]. However, there are few references involving any applications of these passive theorems for singular systems to biological complex systems. Therefore, passivity and its associated control research for biological complex systems are of great interest.

12.1.1 Model Formulation

Consider the physiological process of the endocrine disruptor Diethylstibestrol (DES) moving in human body. The model of this process can be developed in the following Fig. 12.1.

Let $x_1, x_2, x_3,$ and $x_4$ be the quantity of the endocrine disruptor DES in different chambers (organs of human body). Let $k_{ij}$ represent the rate of DES transferring from one chamber to another (from $i$ to $j$, $i, j = 1, 2, 3, 4$). Suppose that $In$ denotes the quantity of intake and that the process of transferring is described by first-order reactions. Based on the above analysis, the differential equation, which follows the conceptual model, is established as follows:

$$
\begin{align*}
\dot{x}_1(t) &= In_1(t) + k_{21}x_2 + k_{31}x_3 + k_{41}x_4 - k_{12}x_1 - k_{13}x_1 - k_{14}x_1, \\
\dot{x}_2(t) &= k_{12}x_1 - k_{21}x_2, \\
\dot{x}_3(t) &= In_3(t) + k_{13}x_1 - k_{31}x_3, \\
\dot{x}_4(t) &= k_{14}x_1 - k_{41}x_4 - k_{45}x_4.
\end{align*}
$$

(12.1)
Fig. 12.1 Poly-chambers conceptual model of the process of endocrine disruptor DES transferring in the human body (1 B1: circulation systems and blood; 2 Ut: reproduction system and uterus; 3 Li: digestive system and liver; 4 Ki: excretion system and kidney; 5 En: environment outside body; In: ingesting and injection).

A medicinal filler to deliver the DES is designed to simultaneously contain two types of drugs. The initial values at different time points are given, and they satisfy the following formulas:

\[ \text{In}_1(t) = \frac{1}{360} t, \quad \text{In}_3(t) = \frac{1}{120} t, \quad t = (0, 1, 2, \cdots, 360), \]

hence \( \text{In}_3(t) = 3 \text{In}(t) \). Suppose that \( u(t) = \text{In}_1(t) = \frac{1}{360} t \), then model system (12.1) can be rewritten as

\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{pmatrix} = 
\begin{pmatrix}
-k_{12} - k_{31} - k_{24} & k_{21} & k_{31} & k_{41} \\
 0 & -k_{21} & 0 & 0 \\
 0 & k_{13} & -k_{31} & 0 \\
 0 & 0 & -k_{45} - k_{41}
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{pmatrix} + 
\begin{pmatrix}
1 \\
0 \\
3 \\
0
\end{pmatrix} u(t).
\]

(12.2)

For convenience, let

\[
A = 
\begin{pmatrix}
-k_{12} - k_{31} - k_{24} & k_{21} & k_{31} & k_{41} \\
 0 & -k_{21} & 0 & 0 \\
 0 & k_{13} & -k_{31} & 0 \\
 0 & 0 & -k_{45} - k_{41}
\end{pmatrix},
B = 
\begin{pmatrix}
1 \\
0 \\
3 \\
0
\end{pmatrix},
\]

and \( x = (x_1, x_2, x_3, x_4)^T \).

According to differential equation models (12.1) and (12.2), the state equation and output equation can be expressed in the simple form,

\[
\begin{cases}
\dot{x}(t) = Ax(t) + Bu(t), \\
y(t) = Cx(t),
\end{cases}
\]

(12.3)

where \( C = \begin{pmatrix} 0 & 0 & 0 & k_{45} \end{pmatrix} \).

According to singular system theory [5], if the dimension of the output variable is identical to that of the input variable, then model system (12.3) can take the following form: