Chapter 4
Bayesian-Based Decision-Making Strategy

4.1 Problem Setup

To illustrate the ideas while avoiding additional computation complexities, in this and the subsequent chapters, it is assumed that there is a single autonomous vehicle performing the search and classification tasks under the probabilistic frameworks. This reflects the case of extremely limited sensing resources, i.e., a single autonomous vehicle as opposed to cooperative MAVs. The extension to MAV decision-making can follow the formulation presented in Section 2.4.5 via sensor fusion. Section 5.4 in Chapter 5 discusses the extended application of risk-based sequential decision-making to the Space Situational Awareness (SSA) problem using a Space-Based Space Surveillance (SBSS) system, which consists of both ground-based sensors and orbiting satellites.

For both the search and classification processes, the Bernoulli-type limited-range sensor model (2.31, 2.32) in Section 2.4.1 is used, however, with different observation contents: \( X(\tilde{c}) = 0 \) for object “present” and \( X(\tilde{c}) = 1 \) for object “absent” in search, and \( X_c(p_k) = 0 \) for object \( O_k \) having property ‘F’ and \( X_c(p_k) = 1 \) for object \( O_k \) having property ‘G’ in classification. Here, an object can be assigned as many property types as needed, but without loss of generality, it is assumed that an object can have one of two properties, either Property ‘F’ or Property ‘G’. Let \( Y_c(p_k) \) be the corresponding classification observation random variable, where \( Y(p_k) = 0 \) corresponds to the observation indicating that there is an object \( O_k \) with property ‘F’ present at position \( p_k \) and \( Y(p_k) = 1 \) corresponds to property ‘G’, respectively. The actual observation is taken according to the probability parameter \( \beta_c \) of the Bernoulli distribution. The general conditional probability matrix \( B_c \) for the classification process is then given as follows

\[
B_c = 
\begin{bmatrix}
\text{Prob}(Y_c(p_k) = 0|X_c(p_k) = 0) = \beta_c & \text{Prob}(Y_c(p_k) = 0|X_c(p_k) = 1) = 1 - \beta_c \\
\text{Prob}(Y_c(p_k) = 1|X_c(p_k) = 0) = 1 - \beta_c & \text{Prob}(Y_c(p_k) = 1|X_c(p_k) = 1) = \beta_c
\end{bmatrix}
\]
Similar as in Section 2.4.1, two types of sensor models can be assumed for classification. For the unit-range sensor model, $\beta_c$ is set as a constant value. For the limited circular range sensor model, the following example is in a same fashion as Equation (2.32),

$$
\beta_c(s) = \left\{ \begin{array}{ll}
\frac{M_c}{r_c} (s^2 - r_c^2) & \text{if } s \leq r_c \\
bn & \text{if } s > r_c
\end{array} \right.,
$$

where $M_c + bn$ is the maximum sensing capability, $s = \|q(t) - p_k\|$, $k = 1, 2, \cdots, N_o$, and $r_c$ is limited classification sensory range. When an object of interest is within the sensor’s effective classification radius $\tilde{r}_c < r_c$, this object is said to be found, and the vehicle has to decide whether to classify it or continue searching. Bayes’ rule is employed to update the probability of object presence at cell $\tilde{c}$ for the search process. Similar as Equations (2.34) and (2.33), we use Bayes rule to update the probability of a found object $O_k$ having property ‘G’ for the classification process, i.e., $P_c(X_c(p_k) = 1)$. Based on the updated probability of object existence, define an information entropy function $H_s(P_{H_s}, \tilde{c}, t)$ as Equation (2.35) as a measure of uncertainty for the search process. For the classification process, define a similar information entropy function $H_c(P_{H_c}, p_k, t)$ as Equation (2.35) for every found object $O_k$ to evaluate its classification uncertainty:

$$
H_c(P_{H_c}, p_k, t) = -P_c(X_c(p_k) = 0) \ln P_c(X_c(p_k) = 0) - P_c(X_c(p_k) = 1) \ln P_c(X_c(p_k) = 1),
$$

where the probability distribution $P_{H_c}$ for the classification process is given by $P_{H_c} = \{P_c(X_c(p_k) = 0), P_c(X_c(p_k) = 1)\}$. There are as many scalar $H_c$’s as there are found objects $O_k$ up to time $t$. The initial value for $H_c$ for every found object $O_k$ can also be set as $H_c = H_{c,\text{max}} = 0.6931$.

### 4.2 Task Metrics

This section develops metrics to be used for the search versus classification decision-making process. For the search process, a same metric as Equation (2.47) is presented when applied to a single vehicle sensor. In the event of object detection and a decision not to proceed with the search process, but, instead, to stop and classify the found object, the associated cost is defined as

$$
\mathcal{J}(t) = \frac{\sum_{c \in \varnothing} H_s(P_{H_s}, \tilde{c}, t)}{H_{s,\text{max}} A_\varnothing}.
$$