There exists a great variety of decision problems concerning allocation and scheduling in a complex of operations (a complex operation system), with many applications to manufacturing and computer systems (see e.g. [8]). This chapter is concerned with the control of the complex of parallel operations containing unknown parameters in the relational knowledge representation. The complex of parallel operations is considered here as a specific uncertain decision plant. The control consists in a proper distribution of the given size of a task or the given amount of a resource, taking into account the execution time of the whole complex. It may mean the distribution of raw material in the case of a manufacturing process or a load distribution in a group of parallel computers. In the deterministic case where the operations are described by functions determining the relationship between the execution time and the size of the task or the amount of the resource, the optimization problem consisting in the determination of the distribution that minimizes the execution time of the complex may be formulated and solved (see e.g. [13]). In the case of uncertainty, various formulations of decision problems adequate for the different descriptions of the uncertainty may be considered [15, 16, 49, 51, 54].

13.1 Complex of Parallel Operations with Relational Knowledge Representation

In the deterministic case the complex of parallel operations is described by a set of functions

\[ T_i = \varphi_i(u_i), \quad i = 1, 2, \ldots, k, \]  

(13.1)

\[ T = \max \{T_1, T_2, \ldots, T_k\} = \max_i \varphi_i(u_i) \triangleq \Phi(u) \]  

(13.2)

where \( T_i \) is the execution time of the \( i \)-th operation, \( u_i \) for each \( i \) is the size of a task in the problem of tasks allocation or the amount of a resource in the problem of resources allocation, \( T \) is the execution time of the whole complex and

\[ u = (u_1, u_2, \ldots, u_k) \in \mathcal{U}. \]
The set $\mathcal{U} \subset \mathbb{R}^k$ is determined by the constraints

$$\bigwedge_{i=1}^{k} u_i \geq 0, \quad \sum_{i=1}^{k} u_i = U$$

(13.3)

where $U$ is the total size of the task or the total amount of the resource to be distributed among the operations. The function $\varphi_i$ is an increasing function of $u_i$ (and $\varphi_i(0) = 0$) in the case of tasks and a decreasing function of $u_i$ in the case of resources. The complex of the parallel operations may be considered as a specific decision (control) plant (Fig. 13.1) with input vector $u$ and a single output $y = T$, described by a function $\Phi(u)$ determined according to (13.2). If the functions $\varphi_i$ are known, it is possible to formulate the following decision (control) problem, optimal from the execution time point of view: to determine the allocation (the distribution) $u^*$ minimizing the execution time $T$, subject to constraints (13.3). It is easy to show that if all operations start at the same time then $u^*$ satisfies the following equations:

$$T_1(u_1) = T_2(u_2) = ... = T_k(u_k).$$

(13.4)

For example, if

$$T_i = c_i u_i, \quad c_i > 0, \quad i = 1, 2, ..., k,$$

(13.5)

then

$$u_i^* = U \left( \sum_{i=1}^{k} \frac{1}{c_i} \right)^{-1}$$

(13.6)

and

$$T^* = \arg \max_{u \in \mathcal{U}} T = U \left( \sum_{i=1}^{k} \frac{1}{c_i} \right)^{-1}.$$

(13.7)