Chapter 3

Asymptotic Stability of Multibody Attitude Systems*

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Abstract: A rigid base body, supported by a fixed pivot point, is free to rotate in three dimensions. Multiple elastic subsystems are rigidly mounted on the rigid body; the elastic degrees of freedom are constrained relative to the rigid base body. A mathematical model is developed for this multibody attitude system that exposes the dynamic coupling between the rotational degrees of freedom of the base body and the deformation or shape degrees of freedom of the elastic subsystems. The models are used to assess passive dissipation assumptions that guarantee asymptotic stability of an equilibrium solution. These results are motivated and inspired by a 1980 publication of R. K. Miller and A. N. Michel [6].

3.1 Introduction

A photograph of the triaxial attitude control testbed (TACT) in the Attitude Dynamics and Control Laboratory at the University of Michigan is shown in Figure 3.1.1. Its physical properties are described in detail in [1], and a detailed derivation of mathematical models for the TACT is given in [2, 3]. The TACT is based on a spherical air bearing that provides a near-frictionless pivot for the base body. The stability problem treated in this chapter is motivated by possible set-ups of the TACT.

This chapter presents results of a study of asymptotic stability properties of an abstraction of the TACT. This abstraction consists of a rigid base body that is free to

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rotate in three dimensions. Multiple elastic subsystems are rigidly mounted on the base body. This chapter discusses the uncontrolled motion of this multibody attitude system. Results are obtained that guarantee asymptotic stability of an equilibrium.

This chapter is motivated by a 1980 publication of R. K. Miller and A. N. Michel [6]. This Miller and Michel paper studied an elastic multibody system consisting of an interconnection of ideal mass elements and elastic springs. Lyapunov function arguments, based on the system Hamiltonian, were used to develop sufficient damping assumptions that guaranteed asymptotic stability of the equilibrium. A key insight was the use of observability properties to guarantee asymptotic stability. The paper by Miller and Michel provided a clear and direct exposition of these issues. It was one of the earliest papers to make clear connections between properties of Hamiltonian systems and their control theoretical properties. During the last 23 years, these issues have been extensively studied. However, the Miller and Michel paper remains an important resource for researchers on dynamics and control of mechanical systems.

3.2 Equations of Motion

Consider the following class of multibody attitude systems: the base body rotates about a fixed pivot point (see Figure 3.2.1). A base body fixed coordinate frame is chosen with its origin located at the pivot point. We assume that the center of mass of the system is always at the pivot point, and thus does not depend on the shape. This is a restrictive assumption, but we demonstrate that it represents an interesting class of multibody attitude systems. Thus gravity does not affect the dynamics and its effects are irrelevant in the subsequent analysis.

The configuration manifold is given by \( Q = SO(3) \times Q_s, \bar{\omega} \in so(3) \) with \( \omega \in \mathbb{R}^3 \) representing the base body angular velocity expressed in the base body frame. We use \( r \in Q_s \) to denote \( n \)-dimensional generalized shape coordinates or deformation