Advanced Trading Strategies

Reduced Monge–Ampère PDEs of Advanced Optimal Portfolio Hedging and Hypoelliptic Obstacle Problems of Optimal Momentum Trading

"... the portfolio-selection process needs to be purged of its reliance on 'static' optimization techniques, which are incapable, by their very nature, of evaluating intertemporal tradeoffs. A fully satisfactory method of portfolio selection must come to grips with the large nonlinear systems of multivariate partial differential equations of dynamic optimality...."


8.1 Remarks

As we have seen so far, mathematics empowered with Mathematica® can be used as a framework for answering many practical questions in the market analysis, investing, and trading of stocks and options. Chapter 5 and 6 present some sophisticated ways as to how to analyze the market from the point of view of estimating the perceived stock volatilities. In Chapter 7 it was shown how mathematics and Mathematica® can be used for synthesizing the available information about market dynamics, market uncertainty, and an investor’s attitude towards uncertainty with respect to explicit trading and investment diversification decisions, provided that the market dynamics is simple enough.

The methodology for determining the implied volatility for European and American options is quite satisfactory. On the other hand, the obvious possible criticism of the stochastic control methods in portfolio management presented so far is that the underlying market dynamics model (the Log-Normal price dynamics), which by virtue of its simplicity has allowed beautiful and efficient symbolic solutions, is quite possibly too simple, and therefore cannot to a significant degree capture and exploit the important features of real life trading. What can be done to improve the
portfolio optimization methodology? Can we extend the methodology described in Chapter 7 to the case where the assumed market dynamics is much more descriptive?

As it turns out, and as it was known for some time, to do this, one would first have to address successfully a fundamental mathematical problem: How to find quality numerical solutions of HJB and Monge–Ampère partial differential equations (in higher dimension)? Since dimension is of paramount significance in numerical solutions of PDEs, we shall show how to reduce (by one) the dimension of a PDE characterizing the value function in the optimal portfolio problem under price dependent price dynamics. We also announce that the problem of finding quality solutions for such Monge–Ampère type PDEs, and therefore also of HJB PDEs that can be reduced to the Monge–Ampère PDEs, has been settled. Those two methodologies combined, the symbolic dimension reduction and numerical solution of reduced equations set the stage, in my opinion, for a new phase in portfolio management technology and practice. We shall illustrate these advances in two computational examples: optimal portfolios of stocks and optimal hedging of options, both under appreciation rate reversing market dynamics.

Finally, as seen in Chapter 7, optimal portfolio hedging requires perpetual trading, which can be afforded only by big investing organizations. For small investors, each trade is paid for, and too frequent portfolio hedging does not seem to be a reasonable alternative. Different kinds of strategies therefore need to be developed. For either type of investors, big or small, one issue that seems worth addressing is: Can we employ advanced mathematics in designing optimal trading strategies in "momentum trading"? To address this problem, the first issue is to introduce a model for "momentum price dynamics", and the question then is: Do we need to move beyond first order scalar SDEs in Markov processes stock-price-dynamics modeling? Specifically, we were led to the second order equation, or which is the same, to a degenerate system of two first order SDEs, as a model for stock-price dynamics.

Finding optimal strategies in such a framework, as it turned out, has lead us again to the fundamental mathematical problem: How to find quality numerical solutions of hypoelliptic problems, and in particular, of hypoelliptic obstacle problems (where, at least in practical terms, the hypoellipticity of generators of Markov processes can be thought of as a precise framework for addressing the issue of degeneracy)? We announce in this closing chapter that this problem has been settled as well.

8.2 Reduced Monge–Ampère PDEs of Advanced Portfolio Hedging

8.2.1 Advanced Optimal Portfolio Hedging Problems

8.2.1.1 The Fundamental Trichotomy

The common feature in portfolio management problems solved so far has been the asset-price-dynamics independence of the "state variable", i.e., of the asset-prices themselves. Similarly, as in option pricing/hedging, in portfolio hedging, time-dependency is much easier to handle than the state, i.e. price-dependency. Therefore we shall refer to portfolio management problems that involve asset-price dynamics which are price-dependent (more precisely, appreciation rates and volatilities are allowed to be price-dependent) as advanced optimal portfolio management problems.