ON THE CENTRAL LIMIT THEOREM FOR MULTIPARAMETER STOCHASTIC PROCESSES

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1. INTRODUCTION AND RESULTS In recent papers Bezandry and Fernique (1990, 1992), Fernique (1993) have given new convergence and tightness criteria for random processes whose sample paths are right-continuous and have left-limits. These criteria have been applied by Bezandry and Fernique, Bloznelis and Paulauskas to prove the central limit theorem (CLT) in the Skorohod space $D[0, 1]$.

In this paper, using recent technique of Bezandry and Fernique, we improve some results of Bickel and Wichura (1971) on weak convergence and tightness for multiparameter processes. The main results of the paper deals with stochastically continuous processes and may be viewed as an extension to multidimensional case of the weak convergence criteria due to Bezandry and Fernique (1990, 1992) and of the CLT due to Bloznelis and Paulauskas (1993), Fernique (1993).

Let $X, X_1, X_2, ...$ be i.i.d. random processes with sample paths in Skorohod space $D_k \equiv D([0, 1]^k, R)$. For details about the space $D_k$ endowed with the Skorohod topology we refer to Neuhaus (1971) and Straf (1972). Denote $S_n = n^{-1/2}(X_1 + ... + X_n - nEX)$. A random process $X$ is said to satisfy the CLT in $D_k$ ($X \in CLT(D_k)$) if the distributions of $S_n$ converge weakly to a Gaussian distribution on $D_k$. For a random process $X = \{X(t), t \in [0, 1]^k\}, k \geq 1$ define

$$\Delta_{(a, b)}^{(i)}X(u) = X(u_1, ... u_{i-1}, b, u_{i+1}, ... , u_k) - X(u_1, ... , u_{i-1}, a, u_{i+1}, ... , u_k),$$

$$u = (u_1, ... , u_k) \in [0, 1]^k, \quad 1 \leq i \leq k, \quad a, b \in [0, 1].$$

A rectangle $B$ in the unit cube $T \equiv [0, 1]^k$ is a subset of $T$ of the form

$$(s, t) = \prod_{i=1}^{k} [s_i, t_i], \quad s = (s_1, ..., s_k), \quad t = (t_1, ..., t_k) \in T;$$

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the i-th face of $B$ is $\prod_{j \neq i} (s_j, t_j)$. Disjoint rectangles $B$ and $C$ are neighbours if they abut and have the same i-th face for some $i$. For a rectangle $B=(s,t]$ let

$$X(B) = \Delta^{(1)}_{(s_1, t_1)} \cdots \Delta^{(k)}_{(s_k, t_k)} X(u)$$

be the increment of $X$ around $B$; $X(\cdot)$ is a random finitely additive function on rectangles. The set $LB(T) = \{(t_1, \ldots, t_k) \in T : t_i = 0 \text{ for some } i\}$ is called the lower boundary of $T$, and $UB(T) = \{(t_1, \ldots, t_k) \in T : t_i = 1 \text{ for some } i\}$ is called the upper boundary of $T$.

**THEOREM 1.** Let $p, q \geq 2$ and $k \geq 1$. Let $X = \{X(t), \ t \in T\}$ be a random process with $EX(t) = 0$, $EX^2(t) < \infty$ for each $t \in T$. Assume $X$ vanishes along the lower boundary of $T$, i.e.,

$$P( X(t) = 0 ) = 1 \text{ for all } t \in LB(T). \quad (1.1)$$

Assume there exist nondecreasing non-negative functions $f, g$ and finite measures $F, G$ on $T$ with continuous marginals such that for all neighbouring rectangles $B, C \subset T$

$$E(| X(B) | \wedge | X(C) |)^p \leq f(F(B \cup C)), \quad (1.2)$$

$$E | X(B) |^q \leq g(G(B)) \quad (1.3)$$

and for some $\varepsilon > 0$

$$\int_0^\varepsilon (u)^{-1-1/p} f^{1/p}(u) log^{k-1}(u^{-1})du < \infty, \quad (1.4)$$

$$\int_0^\varepsilon (u)^{-1-1/(2q)} g^{1/q}(u) log^{k-1}(u^{-1})du < \infty. \quad (1.5)$$

Then $X$ has a version $X'$ with sample paths in $D_k$ and $X' \in CLT(D_k)$.

Note that the random process $X$ is stochastically continuous by (1.3), (1.5).

For one-parameter processes ($k = 1$) Theorem 1 coincides with Th.2 of Bloznelis and Paulauskas (1993b), see also Fernique (1993). Condition (1.1) which appeared yet in Chentsov (1956) and Bickel and Wichura (1971) (and is restrictive for $k \geq 2$)