Chapter 23

Chaos in the Fractal Arteriolar Network

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Why Should the Arteriolar Network Be Fractal?

There is no sure answer to such a philosophical question, but it is a way of phrasing the question of whether or not it might be beneficial to the overall system to have an arteriolar network that is fractal in structure and chaotic in behavior. Remembering that there is no such thing as proof that a model is a correct representation of the system, and that only disproof is possible, we can regard fractal models and chaotic dynamic behavior as potential models for the system, among other models. When are they good models? Why might it be advantageous to the system to be fractal or to have unpredictable behavior?

Fractals are for correlation. Whereas the word derives from Mandelbrot's definition from the Latin fractere, to fragment, the whole story of fractals and chaotic behavior is a story of correlation in space and time. It makes sense that a biological system is not random, but correlated in many features and functions, and this is the main tenet of this chapter.
Parsimony in the instruction set for growth, the genes, is a prime reason for considering whether or not simple recursion rules not only give efficiency, but are adequate to the task. The recursion rule is a repetitive operation: Do something, take the result of that and do something again to it, repeat until the task is accomplished. The rule can be deterministic, as in taking successive powers of two, or probabilistic. A probabilistic rule might be instead of multiplying by exactly two on the next iteration to multiply by two plus or minus some variation. Thus, when one starts with one and generates a set of numbers by multiplying by a number that is something around two, one develops a family of numbers that are not exactly predictable, and never exactly alike but are similar to each other. Furthermore, they fulfill the fractal requirement of being “self-similar,” even if not exactly. The self-similarity is that in going from any one generation of a recursive rule to the next generation, the magnitude approximately doubles. This idea of self-similarity is a hallmark of both fractal structures and chaotic behaviors.

We have only a few genes, on the order of 100,000. In the heart, on which I focus, there are \(10^8\) muscle cells and \(3 \times 10^9\) endothelial cells, and the vascular patterns are complex, so it is obvious that only a few of the 100,000 genes can be used as the instruction set for vascular growth because otherwise there would be none available for guiding the growth of even more complex structures, such as the limbs or the brain. We shall show later that a simple single recursion rule can provide for the design of a vascular network that will fill two- or three-dimensional space. Further, one of the interesting features of fractals is that there is a kind of automatic self-regulation that reduces variation and tends toward uniformity.

**Fractals In Vascular Anatomy**

The arterial network is composed of a series of segments of cylindrical tubes joined at branch points. Downstream, daughter branches are smaller than the parent. Many vascular beds show anastomoses or arcades, but the importance of these varies from tissue to tissue and from animal to animal. In the hearts of pigs and humans, the coronary artery system is basically one of dichotomous branching, two daughters from a parent, repeated recursively