The Problem of Control Synthesis for Uncertain Systems: Ellipsoidal Techniques

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Introduction

This paper deals with a constructive technique of solving the problem of control synthesis under unknown but bounded disturbances in such a way that allows an algrithmization with an appropriate graphic simulation. The original theoretical solution scheme taken here comes from the theory introduced by N. N. Krasovski [1], from the notion of the "alternated integral" of L. S. Pontriagin [2] and the "funnel equation" in the form given in [3]. The theory is used as a point of application of constructive schemes generated through ellipsoidal techniques developed by the authors. A concise exposition of the latter is the objective of this paper. A particular feature is that the ellipsoidal techniques introduced here do indicate an exact approximation of the original solutions based on set-valued calculus by solutions formulated in terms of ellipsoidal valued functions only.

1 The Problem of Control Synthesis

Consider a controlled system

$$(1.1) \quad \dot{x}(t) = A(t)x(t) + u(t) - v(t), \qquad x(t) \in \mathcal{R}^n, \quad u(t), v(t) \in \mathcal{R}^n$$

$$t_0 \leq t \leq t_1$$

with control parameters $u(t)$ subjected to a constraint

$$u(t) \in \mathcal{P}(t)$$

and disturbance $v(t)$ which is unknown but bounded, subjected to a constraint

$$v(t) \in \mathcal{Q}(t).$$

Here $\mathcal{P}(t), \mathcal{Q}(t)$ are *multivalued* maps with values in conv $\mathcal{R}^n$ – the set of all convex compact subsets of $\mathcal{R}^n$. The $(n \times n)$-matrix $A(t)$ is assumed to be continuous.

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The system (1.1) under discussion is an *uncertain system* since its *input* $v = v(t)$, or $v = v(t, x)$, is taken to be *unknown* in advance. The complete information on the state space vector x is assumed to be given at each instant of time t with no bias. Therefore we presume that for each $t \in [t_0, t_1]$ the available information is the *position* $\{t, x_t\}$, $(t \in [t_0, t_1],\ x_t = x(t))$ of the system and also the functions $A(t), \mathcal{P}(t), \mathcal{Q}(t)$ of which the last two are multivalued.

Let $\mathcal{M} \in \operatorname{conv} \mathcal{R}^n$ be a given set. The problem of control synthesis under the informational conditions of the above will consist in specifying a set-valued function $\mathcal{U} = \mathcal{U}(t, x)$, $(\mathcal{U}(t, x) \subset \mathcal{P}(t))$ – "the synthesizing control strategy" which *for any admissible realization* $v(t)$ of the (unknown) parameter v, $v(t) \in \mathcal{Q}(t)$ would ensure that all the solutions $x(t, \tau, x_\tau) = x[t]$ to the equation

$$\dot{x}(t) = A(t)x(t) + \mathcal{U}(t, x(t)) - v(t), \qquad t_0 \leq t \leq t_1 \tag{1.2}$$

that start at a given position $\{\tau, x_\tau\}$, would reach the terminal set $\mathcal{M}$ at the prescribed instant of time $t = t_1$ – provided $x_\tau \in \mathcal{W}(\tau, \mathcal{M})$. Here $\mathcal{W}(\tau, \mathcal{M})$ is the *solvability set* for the problem, namely the set of all those states x_τ from which the solution to the problem does exist in a given class $\mathbf{U}$ of strategies $\mathcal{U}(t, x)$.

The set $\mathcal{W}(\tau, \mathcal{M})$ is the "largest" set (with respect to inclusion) from which the problem is solvable.

We further presume

$$\mathcal{W}[\tau] = \mathcal{W}(\tau, \mathcal{M}) \neq \emptyset, \qquad \forall \tau \in [t_0, t_1].$$

The strategy $\mathcal{U}(t, x)$ will then be selected in a class $\mathbf{U}$ of *feasible feedback strategies* which would ensure that the synthesized system – a *differential inclusion*

$$\dot{x}(t) \in A(t)x(t) + \mathcal{U}(t, x(t)) - \mathcal{Q}(t), \tag{1.3}$$

- does have a solution that starts at any point $x(t_0) = x_{t_0} \in \mathcal{R}^n$ and is defined throughout the interval $[t_0, t_1]$.

The aim of the solution to the problem of control synthesis will now be to find a solution strategy $\mathcal{U}(t, x)$ such that all of the trajectories $x[t] = x(t, t_0, x_{t_0})$ of the differential inclusion (1.3) that start at an initial point $x_{t_0} \in \mathcal{W}[t_0]$, would satisfy the inclusion

$$x(t) \in \mathcal{W}[t], \qquad t_0 < t \leq t_1, \tag{1.4}$$

whatever is the point $x_{t_0} \in \mathcal{W}[t_0]$.

As we shall see in the sequel, the strategy $\mathcal{U}(t, x)$ can be constructed on the basis of $\mathcal{W}[t]$ provided the latter is calculated in advance. The calculation of the set-valued function $\mathcal{W}[t]$, (*the solvability tube*) is therefore a crucial point in finding the overall solution $\mathcal{U}(t, x)$.