Index of Reflection and Refraction

In this chapter we want to derive the reflection and refraction laws of optics from Maxwell's equations. For this purpose, we will consider the behavior of an electromagnetic wave at the interface between two distinct media. We will assume that the media are not ferromagnetic and have a constant polarizability, permeability, and conductivity.

Normal incidence

First, we will consider plane electromagnetic waves striking the interface in the normal direction. From the elementary laws of geometric optics we know that part of the wave is reflected and part of the wave is refracted.

We want to compute the reflection coefficient for a wave. For this purpose, we start from the conditions for continuity for the normal and tangential components of the fields. The distinct media are denoted by the indices 1 and 2.

From Maxwell's equations
\[ \text{div } D = 4\pi \rho \]
\[ \text{div } B = 0 \]

neglecting the surface charge, earlier we found the boundary conditions
\[ D_n^{(1)} = D_n^{(2)}, \quad B_n^{(1)} = B_n^{(2)} \quad (17.1) \]

(The first equation follows from equation (6.6) with the assumption of vanishing surface charge, the second one from equation (11.16).)

Remark \( B_n \) is not the normal component of the \( B \)-field (17.1) which is continuous between media (1) and (2), but is normal to \( da \), see Figure 17.1. The Maxwell equations from which we had concluded the continuity of the tangential components \( E_t \) and \( H_t \) contain now further
terms, so that the validity of the boundary conditions still must be verified. The equation
\[ \text{curl } E = -(1/c)(\partial / \partial t)B \]
yields the integral relation
\[ \int_{\text{area}} \text{curl } E \cdot n \, da = -\frac{1}{c} \frac{\partial}{\partial t} \int_{\text{area}} B \cdot n \, da \]

The figure shows the position of the surface of integration \( a \) lying normal to the interface between the media. With help of Stokes' theorem we obtain from the left-hand side of the equation in the limit \( \Delta y \to 0 \)
\[ \int_{\text{area}} \text{curl } E \cdot n \, da = \oint_{C} E \cdot ds = (E_{t}^{(1)} - E_{t}^{(2)}) l \]
and for the right-hand side, as \( \Delta y \to 0 \),
\[ \frac{\partial}{\partial t} \int_{\text{area}} B \cdot n \, da = \frac{\partial}{\partial t} B_{n} l \Delta y \to 0 \]
Thus, we obtain the continuity of the tangential components

\[ E_{t}^{(1)} = E_{t}^{(2)} \quad (17.2) \]

Correspondingly, from the Maxwell equation

\[ \text{curl } H = \frac{1}{c} \frac{\partial}{\partial t} D + \frac{4\pi}{c} j \]
assuming that no surface currents are present, in an analogous calculation we obtain the boundary condition

\[ H_{t}^{(1)} = H_{t}^{(2)} \quad (17.3) \]

Having been convinced of the validity of the former boundary conditions we want to investigate now the following special case: let a wave be incident from the vacuum on a medium with the generalized index of refraction \( p = p_{1} + ip_{2} (16.16) \). Let the \( (y,z) \)-plane be the interface, as in Figure 17.2. With this simplification the wavevector is \( k = ke_{x} \), and also \( k \cdot r = kx \). We suppose the incident wave to be a plane wave:

\[ E_{y} = ae^{i(kx-\omega t)}, \quad H_{z} = be^{i(kx-\omega t)} \quad (17.4) \]