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Random Numbers: What They Are and How to Use Them

The lot causeth disputes to cease, and it decideth between the mighty.

_Proverbs_ 18:18

13.1 What are random numbers?

In 1955, the Rand Corporation published a book listing a million random digits (see [27]). A typical page contains hundreds of the digits 0 through 9 written for easy reference in little matrices (squares) having five digits along each row and each column. We are interested in two main questions: what are random numbers and why do we need them?

The most basic way to define random numbers is that they are numbers generated by a random procedure involving repeated independent trials. When we speak about the random digits 0 through 9 it is assumed that a trial of the procedure yields each of the ten digits with probability .1. Here is how we might generate these random digits. Suppose we take ten cards of the same size and write the digits 0 through 9 on the cards so that each card has a different digit. Then take a large hat, say, toss in the cards, and mix well. Now choose a card at random from the hat, that is, reach in without looking and choose. Write down on a piece of paper the digit appearing on the card you have chosen. Put the card back into the hat and mix the cards again. Repeat the procedure by choosing a card at random, writing down the digit appearing on the card, replacing, mixing, choosing again, and so on. The string of digits you are writing down constitutes a string of
random digits because it has been produced by a random device supposed to yield each digit with probability \( .1 \) in independent trials (the procedure might not really give digits with equal probability because of poor mixing or other reasons—see, e.g., Section 13.6). You could have produced the random digits by an equivalent method by using, for instance, a modified roulette wheel in which the wheel has been divided into ten equal parts, each one corresponding to one of the ten digits. The book of random digits published by the Rand Corporation was produced by a more sophisticated version of this roulette wheel, one in which electronic pulses are used instead of spinning disks.

Given random digits, how do we get more complicated random numbers? Suppose you had generated the sequence 3217900597 by reading ten digits from the table. Then each digit is random, and furthermore the two-digit numbers 32, 17, 90, 05, 97 obtained by taking the numbers two at a time are random numbers because they have been produced by a random procedure ensuring that the one hundred two-digit numbers 00 through 99 each have probability \( .01 \) of appearing and, moreover, the selections of these two-digit numbers are independent. This follows from the way we selected the individual digits, namely, in a uniform, independent manner. Thus, taking the original ten-digit sequence and choosing the numbers two at a time going backward to get 79, 50, 09, 71, 23 also give random two-digit numbers, as does any other way you might think of to get two-digit numbers using the generated string, as long as the method does not use the same selection more than once. If you need five two-digit random numbers you can use any of the methods to generate them. None of these methods is any better than any other, so you may as well take the simplest one: 32, 17, 90, 05, 97, reading from left to right.

Note that if you need ten random two-digit numbers, which requires twenty digits all together, you should select 20 random digits from the table, not use, for example, 32, 17, 90, 05, 97, 79, 50, 09, 71, 23 by selecting digits read both forward and backward from the same initial ten-digit string. Randomness is lost when the same digits are used more than once in this way. This is evident above because the first five two-digit numbers give us information about the whole sequence, (for instance, that 6 does not occur), and so following selections are neither equally likely nor independent.

For the same reason, we would not want to use the same lines or the same page of the random number table over and over again every time we need random numbers. If we do this, the unpredictability associated with randomness is lost. With the Rand table you have a million digits on many pages. You should use this abundance to strive for unpredictability, that is, randomness. One way you could select digits from the Rand book is by starting at the first page and reading line by line (or column by column) until you have enough digits, then marking where you stop. The next time you need digits, begin reading where you left off the first time, and so on. In this way, you will go through the book. Another method starts by using