The Need for the Infinite

Parmenides

In opposition to Anaximander, Parmenides of Elea, Italy (480 B.C.) was a monist. That is, he held that the universe consists of only one object. The number of things that exists is just one.

The unique thing, according to Parmenides, does not have infinite duration, but exists timelessly, and changelessly: ‘nor was it, nor will it be, since now it is, all together, one’. Nor does the one existing object have infinite spatial extension: ‘it is completed on all sides, like the bulk of a well-rounded ball’. (The quotations are from J. Barnes, Early Greek Philosophy, pages 134-5.)

Parmenides taught that nothing moves, since motion implies the existence of more than one thing, namely, a finishing place and a starting place. Although it may look as if something is moving, this is just an illusion.

Zeno

Zeno (450 B.C.) was a disciple of Parmenides. He produced four arguments for the conclusion that there is no motion — this in support of the claim of his master.
Zeno’s First Argument

Motion is impossible, said Zeno, because a moving object must first go half the total distance it will travel, then half the remaining distance, and so on, forever. If a point moves from position 0 to position 1 on the number line, it first reaches position $1/2$, then position $3/4$, then position $7/8$, and so on. At the $n$th stage, it is at position $1 - \frac{1}{2^n}$. From the fact that there is no $n$ such that $1 - \frac{1}{2^n} = 1$, it follows that that moving point never reaches position 1. It just cannot get through the infinite number of stages necessary to do so. Hence there is no motion, motion from 0 to 1 being typical of any motion whatsoever.

In modern physics, we counter this argument by asserting that, indeed, the point can and does traverse each of the infinite number of intervals from $1 - \frac{1}{2^n}$ to $1 - \frac{1}{2^{n+1}}$ for $n = 1, 2, 3, \ldots$ — ad infinitum. There is no $n$ such that the moving point does not cross position $1 - \frac{1}{2^n}$. Starting from the premiss that there is motion, modern physicists invoke the infinite to explain it. Like Zeno, they assume that motion is continuous, but, unlike Zeno, they are willing to say that a moving object does pass over an infinite number of points. Zeno rejected the infinite, and so he rejected motion too. Modern physicists accept motion, and so they accept the infinite too.

Zeno’s Second Argument

The famous runner Achilles and his rival (usually thought to be a tortoise) are racing along the positive number line. Achilles starts at position 0, but the tortoise has a head start, beginning at position 1. Since Achilles runs twice as fast as the tortoise, one might expect him to overtake the tortoise at position 2. However, when Achilles arrives at position 1, the tortoise is already at position $1 + \frac{1}{2}$; when Achilles reaches position $1 + \frac{1}{2}$, the tortoise has raced on to position $1 + \frac{1}{2} + \frac{1}{4}$; and so on. When Achilles finally gets to position $2 - \frac{1}{2^n}$, for large $n$, the tortoise is still ahead, at position $2 - \frac{1}{2^{n+1}}$. Despite the appearances, which lead us to believe there is motion, Achilles will never catch up to the tortoise.

In this second argument, Zeno again assumed, as we do, that space and time are continuous, and that, if there is motion, there is uniform motion. Zeno also assumed, unlike us, that Achilles and the tortoise can never ‘get through’ the infinite number of stages into which Zeno analysed their motion.

For modern physics, precisely, motion typically consists of the occupation of infinitely many distinct locations at infinitely many distinct instants — all within a finite time interval. Because we accept the infinite, we do not find Zeno’s argument troubling. However, if someone rejected the infinite, he or she would, indeed, have to reject the possibility of continuous motion.