CHAPTER 3

Noncanonical Linear Programming Problems

§0. Introduction

The simplex algorithm discussed in Chapter 2 solves canonical maximization and canonical minimization linear programming problems. The important properties that characterize a canonical linear programming problem (in this book at least) are the nonnegativity of the initial independent variables and the inequality form of the main constraints. However, easy modifications of the algorithms of Chapter 2 enable the solution of certain noncanonical linear programming problems. The concern of this chapter is the formalization of these modifications. Our linear programming solution procedure will consequently apply to a broader class of problems. In addition, the solution of the noncanonical problems here will be crucial to our first application in Chapter 5.

§1. Unconstrained Variables

Definition 1. A real variable in a linear programming problem is said to be unconstrained if there is no nonnegativity constraint on the variable.

The first type of noncanonical linear programming problem has canonical maximization or canonical minimization form except that there may not be nonnegativity constraints on all of the independent variables, i.e., some of these variables may be unconstrained. Fortunately, such a problem is easily transformed into an equivalent linear programming problem in canonical form plus a number of filed equations. We illustrate with several examples.
EXAMPLE 2.  

Maximize \( f(x, y) = x + 3y \)  
subject to  
\[
\begin{align*}
  x + 2y & \leq 10 \\
  -3x - y & \leq -15.
\end{align*}
\]

In this problem, both \( x \) and \( y \) are unconstrained. Before we illustrate the solution procedure, we sketch the constraint set of this problem:

On the basis of this constraint set, can you guess the outcome of the problem?  

We begin by recording the problem in a Tucker tableau as usual. To record the fact that \( x \) and \( y \) are unconstrained, we circle those variables.

Note that slack variables are always constrained to be nonnegative. Our goal now is to pivot each unconstrained independent variable down to the east. Since the tableau represents a noncanonical linear programming problem, the simplex algorithm of Chapter 2 is not used for this. The simplex algorithm only applies to canonical linear programming problems! The acceptable pivot entries are \( a_{11}, a_{12}, a_{21}, \) and \( a_{22}; \) in a noncanonical maximum tableau (or canonical maximum tableau for that matter), never pivot on any entry in the \(-1\) column or the objective function row. The most convenient of these pivots for our purposes is \( a_{11} = 1 \) or \( a_{22} = -1 \) since all tableau entries will remain integral after pivoting. We choose \( a_{11} = 1 \) for definiteness; pivoting on 1 moves \( x \) down to the east: