CHAPTER XII. Two-Fluid Models of Turbulence

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1. INTRODUCTION

1.1 Some defects of turbulence models

It has become customary among engineers and others concerned with practical flow and heat-transfer predictions to use "turbulence models", i.e., sets of equations which purport, when coupled with the equations of mean motion, to describe such statistically significant properties as the "effective viscosity", "length scale" and "energy". Most frequently used are those models employing two partial differential equations, the dependent variables of which are, for example, the turbulence energy, $k$, and the dissipation rate $\varepsilon$.

Although valuable service has been performed by such models, their predictive ability is far from meeting all demands. A brief list of their shortcomings is:

(a) The "intermittency" of turbulent flow, evident to all who look perceptively at, for example, automobile-exhaust plumes, is left out of account.
(b) Attempts to extend the models to chemically-reacting flows have had little success, because chemical reactions take place in regions of steep gradients which find no expression in the equations employed.
(c) Empirical and non-general corrections have had to be made to the "constants" of the equations in order to bring their predictions into line with experimental evidence regarding the influences of mean-streamline curvature and of interactions between gravitational forces and temperature gradients.
(d) The models are totally unable to explain the "unmixing" phenomenon which steepens gradients of average fluid properties rather than diminishing them.

In the view of the present writer, the above defects have a common origin, namely neglect of the "spottiness" of real turbulent flows. The question therefore to be discussed is: How can the major effects of the "spottiness" be described by a mathematical apparatus that is still not too elaborate for practical use?
1.2 A proposed shift of emphasis

(a) The origins

Modern ideas about turbulence have been greatly influenced by the writings of Osborne Reynolds (eg Ref 1) and Ludwig Prandtl (eg Ref 2), whose observations had led them to conceive of turbulence as involving a semi-random exchange of matter, similar to that envisaged in the kinetic theory of gases, but on a larger scale. The following quotations illustrate this.

Reynolds:
"The heat carried off ... is proportional to the rate at which particles or molecules pass backwards and forwards from the surface". (Present author's italics.)

Prandtl:
"It is now necessary to make a usable hypothesis for the mixing velocity, w. The transverse momentum associated with this velocity must be constantly destroyed by braking, and constantly re-established". (Present author's italics).

It is a consequence of this preoccupation with mixing that turbulent fluxes of a scalar quantity are almost invariably expressed, by analogy with Fick's law of diffusion and Fourier's law of heat conduction, in terms of the gradient of the corresponding intensive property. Thus, for heat transfer, the law:

\[
\langle q_d \rangle = -\lambda_{\text{eff}} \cdot \text{grad } T
\]

is employed, along with the supposition that \( \lambda_{\text{eff}} \) is a greater-than-zero measure of the local turbulent motion. Here \( \langle q_d \rangle \) is the heat-flux vector, \( \lambda_{\text{eff}} \) is the effective conductivity, and \( T \) is the time-mean temperature.

So embedded is this notion in current thinking that, when experimental evidence becomes incontrovertible that the so-defined \( \lambda_{\text{eff}} \) is negative, the phrase "contra-gradient diffusion" (Ref 3) is applied to the phenomenon, which is regarded as anomalous.

(b) The sifting phenomenon and its implications

In the view of the present writer, normal, ie "co-gradient" diffusion is only one of the two main ways in which turbulent fluxes are caused; the second, although it could be called "non-gradient diffusion", is better described as "sifting". This is the phenomenon