By about 2000 B.C. the Egyptians knew (or believed) that if a right triangle had legs of lengths 3 and 4 units, then the hypotenuse—the side opposite the right angle—had a length of 5 units.

Later this was generalized by the Babylonians. Babylonian clay tablets dating back to about 1600 B.C. record the theorem \( a^2 + b^2 = c^2 \), relating the lengths of the sides of any right triangle. But the Babylonian mathematicians probably could not prove it.

3.1. The Theorem

This theorem is used today, as it was in ancient times, for laying out square corners of fields and foundations. It is also of vital importance in problems ranging from carpentry and navigation to astronomy and to studies of the very nature of space and time.

The Pythagorean Theorem. If \( a \) and \( b \) are the lengths of the two legs of a right triangle and \( c \) is the length of the hypotenuse, then

\[
a^2 + b^2 = c^2.
\]
As noted above, this theorem was stated without proof around 1600 B.C. But, the first general proof is usually credited to the Greek philosopher Pythagoras (ca. 580–500 B.C.). And the theorem is named after him. (In fact, a real proof of the Pythagorean Theorem may not have been given until Euclid's, 200 years later.)

Many different proofs have been given since then. One of the simplest, described below, was discovered by U.S. Representative James A. Garfield (1831–1881) 5 years before he became the 20th President of the United States.

Garfield’s proof makes use of the expression for the area of a right triangle. The triangle in Figure 1 has area \( \frac{1}{2}ab \) since it is just half of a rectangle with sides of lengths \( a \) and \( b \).

![Figure 1](image_url)

**Proof of the Pythagorean Theorem.** Figure 2 shows three triangles forming half of a square with sides of length \( a + b \). Two of these triangles (shaded) are congruent to our original right triangle. Now note that angles \( A \), \( B \), and \( D \) satisfy the relations

\[
A + B = 90° \quad \text{and} \quad A + B + D = 180°.
\]

So \( D = 90° \), and the third triangle is also a right triangle.

![Figure 2](image_url)

The area of the half square is

\[
\frac{1}{2}(a + b)^2 = \frac{1}{2}(a^2 + 2ab + b^2),
\]