12 A Tailored Suit for Risk Management: Hyperbolic Model

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12.1 Introduction

In recent years the need to quantifying risk has become increasingly important to financial institutions for a number of reasons: the necessity for more efficient controlling due to globalisation and sharply increased trading volumes; management of new financial derivatives and structured products; and enforced legislation setting out the capital requirements for trading activities.

As mentioned in Ridder (1998) “the idea of ‘Value at Risk’ (VaR) reflects the industry’s efforts to develop new methods in financial risk management that take into account available knowledge in financial engineering, mathematics and statistics”. Three standard methods are currently used to evaluate market risk: historical simulation, which in principle is a bootstrap approach, the variance-covariance approach that is also called ‘delta normal method’, and Monte Carlo simulation. For an in-depth presentation of these techniques the reader is referred to Jorion (1998) and Dowd (1998).

Risk, however, is multifaceted, and it has been shown elsewhere (e.g. Artzner, Delbaen, Eber and Heath (1997)) that VaR alone can be deficient in certain regards. A natural property a risk measure is expected to satisfy is subadditivity: the risk of a portfolio should be smaller than the sum of risks associated with its subportfolios. This can also be expressed as: it should not be possible to reduce the observed risk by dividing a given portfolio into subportfolios. In this sense, VaR is not subadditive. For this reason other definitions of risk can and should be used depending on the circumstances.

The entire stochastic uncertainty (risk) that is associated with a particular portfolio for a set time horizon is encapsulated within its P&L distribution \( F(x) \) (Kokic, Breckling and Eberlein (1999)). For any profit \( x \) the
function $F(x)$ gives the probability of obtaining no greater profit than $x$ over the time horizon. Thus the most desirable distribution functions are those which increase most slowly and consequently are depicted below all other curves that represent alternative portfolios.

By reading the appropriate quantile value, $VaR$ can be obtained directly from the P&L function. While for certain operations of a financial institution $VaR$ is a suitable measure of risk, for other operations risk may have to be defined differently. In the classical capital asset pricing approach (Huang and Litzenberger (1988), p.98), for example, risk is measured in terms of standard deviation (or volatility) of the portfolio. The advantage of centering the analysis on the P&L distribution is that all risk measures of interest are just specific functions of $F(x)$.

A fully-fledged risk management system should therefore enable the user to define risk his/her own way as a function of $F(x)$. Moreover, rather than focusing on risk alone, it may be warranted to relate it to chance. The most common measure of chance used in financial analysis is the expected return from a portfolio over a given time frame, although chance could also be defined as the median return, for example, which is a far more robust measure than the mean. These, like any other meaningful measure of chance, can also be expressed in terms of the P&L distribution function. In the context of portfolio management it is equally important to look at the chance side as well as the risk side of the return distribution. Changing a portfolio in order to alter its risk exposure will typically affect the chances as well. How credit risk can be consistently incorporated into this framework, is outlined in section 6, while section 7 presents an example to demonstrate the merits of using the hyperbolic model to describe the P&L distribution.

### 12.2 Advantages of the Proposed Risk Management Approach

Aspects of a modern risk methodology can be summarized as follows: recognition of the fact that risk assessment actually amounts to a forecasting problem; no assumption of a symmetric P&L distribution, enabling an adequate account of fundamental and derivative securities within the same portfolio; consistent treatment of market and credit risk; an explicit account of inter-market dependencies, allowing for conditional risk assessment (e.g. what would happen to risk if the US dollar was to rise by 1 per cent); flexibility to define appropriate summary statistics according to the task that is to be performed; confidence bounds on the model that is being fitted and on all derived summary