Dependence has previously been classified into three time frames: instantaneous, short-term, and long-term (Section 3.2). Almost all the standard models studied until now lead to long-term dependence. Important examples are the Markov multi-state models and the shared frailty models. Only a single standard model, the Marshall-Olkin model of Section 5.5.4, displays instantaneous dependence. No standard models show short-term dependence, even though we have argued that many common subject matter problems possibly or probably display short-term dependence. Therefore, it makes sense to develop models that can be used to discuss whether the dependence is of short-term or long-term time frame. Multi-state models can easily be constructed to model short-term dependence by releasing the Markov assumption, substituting with a Markov extension model as shown in Chapter 5, but they have no interpretation, whereas a frailty model has an interpretation as a random effects model. In particular for frailty models, it is an advantage to have short-term dependence models for checking the fit of the ordinary shared frailty model. The aim of this chapter is to extend the frailty models to describe both instantaneous and short-term dependence.

On the general level, these models are random effects models with three sources of variation. For parallel data, these are group effects and individual random effects, as in the shared frailty model, and besides this, there is a random group by time interaction effect. For recurrent events, the corresponding random effects are individual variation and simple variation, which in this chapter is supplemented with a random variation over time.
This chapter describes the models both for parallel data and recurrent events.

The models of this chapter do require further development and further application before they can be applied routinely. The aim of this chapter is to present the concepts and the ideas and open up a new area of research. This chapter is more difficult to follow and includes fewer ready-to-use expressions than the rest of this book. Consequently, readers who are only interested in the better-established models may prefer to omit this chapter.

A major problem with these models is their mathematical complexity. It is not possible to handle the most interesting models; that is, it is necessary to consider models that are simpler than desired, which is disappointing from a practical point of view. For example, we would prefer a model with the frailty varying in continuous time, but it is easier to study models, where the frailty is piecewise constant. A consequence of this is that the model is not truly a short-term dependence model, within each interval the dependence is rather of long-term time frame. Another point along this line is that we would prefer a multiplicative model rather than an additive, but multiplicative models appear to be more complicated computationally. Also the models considered here will often assume constant hazards in order to simplify the evaluations.

Instantaneous dependence can be obtained by the frailty being modeled as an independent increments process (Section 11.1). This is a continuous time model for a randomly varying environment. In that case, the common risks are independent for disjoint time intervals, implying the possibility of several events’ happening at the same time. In the case of recurrent events, this model can also be derived by means of a subordinated stochastic process. This derivation corresponds to a randomly varying speed of time. Furthermore, this process can be added to a constant frailty process, giving a model containing both the constant frailty model and the independent increments frailty model so that the relevance of both models can be tested in a proper way (Section 11.2).

Short-term dependence is mathematically more complicated to handle than instantaneous dependence, but more relevant in practice. It can be introduced by models similar to the additive models described in Chapter 10, but having the additive structure over time rather than over individuals (Section 11.3). This gives a model with piecewise constant frailty. Section 11.4 describes a moving average model, which is a model intended for reducing the jumps in the dependence in the piecewise constant frailty model of Section 11.3.

Another idea that can be used to generate short-term dependence is a hidden cause of death model (Section 11.5), which is still a shared frailty model. This is based on the idea that each person has a constant, but multivariate frailty, with one coordinate for each cause of death. This makes it possible to have a more smooth change over time. However, this model requires major development before it can be applied in practice.