6 Two-Way Nested (Hierarchical) Classification

6.0 PREVIEW

In Chapters 3 through 5 we considered analysis of variance for experiments commonly referred to as crossed classifications. In a crossed-classification, data cells are formed by combining of each level of one factor with each level of every other factor. We now consider experiments involving two factors such that the levels of one factor occur only within the levels of another factor. Here, the levels of a given factor are all different across the levels of the other factor. More specifically, given two factors $A$ and $B$, the levels of $B$ are said to be nested within the levels of $A$, or more briefly $B$ is nested within $A$, if every level of $B$ appears with only a single level of $A$ in the observations. This means that if the factor $A$ has $a$ levels, then the levels of $B$ fall into $a$ sets of $b_1, b_2, \ldots, b_a$ levels, respectively, such that the $i$-th set appears with the $i$-th level of $A$. These designs are commonly known as nested or hierarchical designs where the levels of factor $B$ are nested within the levels of factor $A$.

For example, suppose an industrial firm procures a certain liquid chemical from three different locations. The firm wishes to investigate if the strength of the chemical is the same from each location. There are four barrels of chemicals available from each location and three measurements of strength are to be made from each barrel. The physical layout can be schematically represented as in Figure 6.1. This is a two-way nested or hierarchical design, with barrels nested within locations. In the first instance, one may ask why the two factors, locations and barrels, are not crossed. If the factors were crossed, then barrel 1 would always refer to the same barrel, barrel 2 would always refer to the same barrel, and so on. In this example, this is clearly not the situation since the barrels from each location are unique for that particular location. Thus, barrel 1 from location I has no relation to barrel 2 from any other location, and so on. To emphasize the point that barrels from each location are different barrels, we may recode the barrels as 1, 2, 3, and 4 from location I; 5, 6, 7, and 8 from location II; and 9, 10, 11, and 12 from location III. For another example, suppose that in order to study a certain characteristic of a product, samples of size 3 are taken from each of four spindles within each of three machines. Here, each
The Analysis of Variance

Locations

Barrels

Measurements

FIGURE 6.1 A Layout for the Two-Way Nested Design Where Barrels Are Nested within Locations.

Machines

Spindles

Samples

FIGURE 6.2 A Layout for the Two-Way Nested Design Where Spindles Are Nested within Machines.

separate spindle appears within a single machine and thus spindles are nested within machines. Again, the layout may be depicted as shown in Figure 6.2.

Both nested and crossed factors can occur in an experimental design. When each of the factors in an experiment is progressively nested within the preceding factor, it is called a completely or hierarchically nested design. Such nested designs are common in many fields of study and are particularly popular in surveys and industrial experiments similar to the ones previously described. In this