A Fully Formalized Theory for Describing Visual Notations

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ABSTRACT This chapter addresses issues in visual language theory with the help of logic formalisms that were developed for reasoning tasks by the artificial intelligence and spatial databases community, especially for spatial and diagrammatic reasoning. We describe an approach based on three formal components. Topology is used to define basic geometric objects. Theory about spatial relations from the domain of spatial databases is employed to define possible relationships between visual language elements. Description logic theory from the AI community is used to combine topology and spatial relations. We prove the feasibility of our theory by describing three representative visual notations: entity-relationship diagrams, petri nets, and a pictorial language for concurrent logic programming.

9.1 Introduction

This chapter reports on an approach to formalizing visual notations. We propose a spatial logic for describing syntax and static semantics of visual notations. This logic combines three components (topology, spatial/topological relations, description logic) that are themselves also formally specified with precise semantics. These components were derived from research communities that are related to visual language (VL) research: reasoning on diagrammatic representations and spatial databases. The goal of this chapter is the attempt to intensify the dialogue between these research communities and to "advertise" the benefits of this particular view of VL theory.

The successful application of our theory to a completely visual language for concurrent logic programming, Pictorial Janus (PJ) [27, 28], has been reported in [19]. A revised and simplified version of PJ's language specification is also presented in this chapter. The experience with PJ resulted in the development of the editor GenEd [23] for designing visual notations. GenEd's generic semantics is based on and controlled by the theory described in this chapter.

GenEd is an object-oriented editor supporting the formal design and analysis of visual notations. Prominent features of GenEd are (1) it is
generic, i.e. domain-specific syntax and semantics of drawings are specified by users; (2) it has a built-in parser for actual drawings, driven by our spatial logic; (3) it offers powerful reasoning capabilities about diagrams and their specification.

Our approach is generic in the sense that particular instances can be chosen for the above mentioned components. This process depends on the nature of specific visual notations to be formally specified. For instance, the definition of PJ is mostly based on topological relations between lines, arrows, and regions. Therefore, we have selected corresponding definitions for primitive geometric objects, an appropriate theory on spatial (topological) relations \([11]\) that can deal with true 1D objects and regions, and a matching description logic. However, we like to emphasize that other visual languages or notations might require different definitions for objects and their possible relationships.

This chapter is organized as follows. The next section discusses the theoretical foundation of our approach. Afterwards we describe the editor GenEd that implements our theory. This is followed by two sections presenting three representative visual notations and their formal specification. We conclude this chapter with a discussion of related work and ongoing research.

9.2 Theoretical Foundation

We believe that the semantics of representational devices used for VL theory should be well understood. That is, the meanings of represented language concepts should be unambiguously determined by explicit notational devices whose meanings (semantics) are understood, so that algorithms can operate on the representation in accordance with the semantics of the notation, without needing ad hoc provisions for specific VL domains. In the following we outline a fully formalized theory for describing visual notations that consists of three components. Each component is defined by precise semantics. The definition of objects and relations is based on point-sets and topology. Description logic theory can be based on model-theoretic semantics appealing to first-order logic or on a compositional axiomatization with set theory. The next sections describe the components in more detail and briefly review alternative instances for these components.

9.2.1 Objects and Topology

The definition of basic geometric objects (the elementary vocabulary of a visual notation) usually relies on topology which is itself a basis for defining relationships between objects. In the following we assume the usual concepts of point-set topology with open and closed sets \([39]\). The interior of