We start this chapter by explaining how to use vectors in Mathematica, with an emphasis on practical operations on vectors in the plane and in space. We discuss the standard vector operations, and give several applications to the computations of geometric quantities such as distances, angles, areas, and volumes. The bulk of the chapter is devoted to instructions for graphing curves and surfaces.

Vectors

In Mathematica, vectors are represented as lists of numbers or variables. You write a list in Mathematica as a sequence of entries encased in braces. Thus, you would enter \( v = \{3, 2, 1\} \) at the prompt to tell Mathematica to treat \( v \) as a vector with \( x, y, z \) coordinates equal to 3, 2, and 1, respectively.

You can perform the usual vector space operations in Mathematica: vector space addition, scalar multiplication, and the dot product. Here are some examples:

\[
\text{In[1]:=} \quad a := \{1, 2, 3\}; \\
b := \{-5, -3, -1\}; \\
c := \{3, 0, -2\}; \\
\text{In[2]:=} \quad a + b \\
\text{Out[2]=} \quad \{-4, -1, 2\}
\]
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In[3]:= \text{5c}
\text{Out}[3]= \{15, 0, -10\}

In[4]:= \text{a.b}
\text{Out}[4]= -14

The period key is used to compute the dot product. (Alternatively, you can write \text{Dot}\{a, b\}. As usual, you can use the dot product to compute lengths of vectors (also known as vector norms).

In[5]:= \text{lengthofa = N[Sqrt[a.a]]}
\text{Out}[5]= 3.74166

Here is a function that automates the numerical computation of vector norms.

In[6]:= \text{norm[v_] := N[Sqrt[v.v]]}
In[7]:= \text{norm[a], norm[b], norm[c]}
\text{Out}[7]= 3.74166
\text{Out}[8]= 5.91608
\text{Out}[9]= 3.60555

Our attention throughout this book will be directed to vectors in the plane and vectors in (three-dimensional) space. Vectors in the plane have two components; a typical example in \text{Mathematica} is \{x, y\}. Vectors in space have three components, like \{x, y, z\}. The following principle will recur: Vectors with different numbers of components don't mix. As you will see, certain \text{Mathematica} commands will only work with two-component vectors; others will only work with three-component vectors. To convert a vector in the plane into a vector in space, you can add a zero to the end.

In[10]:= \text{pl = \{x, y\}}
\text{Out}[10]= \{x, y\}

In[11]:= \text{sl = Join[pl, \{0\}]}  
\text{Out}[11]= \{x, y, 0\}

To project a vector in space into a vector in the \text{x-y} plane, you simply drop the final component.