

Markovian Repairman Problems. Classification and Approximation

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Abstract: Classification of Markov repairable systems are given by the equilibrium point of the velocity drift. The diffusion approximation of the normalized queueing process by the Ornstein–Uhlenbeck process is established.

Keywords and phrases: Repairable system, equilibrium point, diffusion approximation

10.1 Introduction

The repairman problems can be considered as basic models of stochastic systems in the reliability theory. In the same time the repairman problem is the elementary queueing system occurring according to the birth-and-death Markov process as a mathematical model of stochastic system. A Markovian model of a repairman problem is given by the following parameters. There are $n$ identical working devices which operate independently and simultaneously. The working time of every device has the exponential distribution with intensity $\lambda$. There are also $r \leq n$ repairing facilities which serve to repair working devices when they are broken. In the case of $r = n$ a repairman system is a supply energy system [Feller (1958)]. The service time of every facility has the exponential distribution with intensity $\mu$. Besides, there are $m$ spare devices which are used for the replacement of broken devices. In the case of $m = 0$ a repairman system is a system with limited service [Derzko and Korolyuk (1997)]. Therefore, there are at most $n$ devices operating at any time. The queueing process $\nu_n(t)$ in a repairman problem is described by the number of devices undergoing or waiting to be repaired at the moment of time $t$.

Under above formulated condition the queueing process $\nu_n(t)$ is a Markov
jump process with the intensities of jumps given by the following relations. The intensities of jump value +1 are

$$\lambda_n(k) = \begin{cases} n\lambda, & k \leq m \\ (n + m - k)\lambda, & k \geq m \end{cases} \quad (10.1)$$

The intensities of jump value -1 are

$$\mu_n(k) = \begin{cases} k\mu, & k \leq r \\ r\mu, & k \geq r \end{cases} \quad (10.2)$$

The formulae (10.1) and (10.2) completely determine the behaviour of the queueing process in the repairmen problem. This formulae can be used in simulation of trajectories of the queueing process $\nu_n(t)$ applying the computer program based on the stochastic representation of the Markov jump process [Korolyuk and Korolyuk (1997, Section 2.1)].

Note, that the queueing process $\nu_n(t)$ takes integer values in the interval

$$0 \leq \nu_n(t) \leq n + m.$$ 

Therefore, the phase state space of values of the queueing process is the following finite set of integer numbers

$$E_n = \{0, 1, \ldots, n + m\}.$$ 

The boundary states 0 and $n + m$ are reflecting barriers that is meant, that the queueing process $\nu_n(t)$ transpose from state 0 to state 1 and from state $n + m$ to state $n + m - 1$ with the probability 1.

Hence, the queueing process $\nu_n(t)$ has the steady-stable distribution. But, it is evident, that the calculation of this stationary distribution can be realized only in some very particular cases. It is evident also, that the calculation of the global characteristics of behaviour of the queueing process on the large enough time interval can be realized by using simulation technique only for a fixed real valued initial parameters $n, m, r, \lambda$ and $\mu$ which determine the queueing process.

The more effective approach in analysis of the repairman problem is an asymptotical investigation of the queueing process $\nu_n(t)$ as $n \to \infty$. As usually in the mathematical application the limit behaviour of mathematical model can be used effectively for real stochastic system which is determined finite values of the series parameters.

As it is well-known, the limit behaviour of the queueing process as usually is described by application of the average and diffusion approximation schemes. In the repairman problems there are such results but, as it is seems to us, there are another models which can be investigated in the asymptotical analysis.

The various situations can be considered under different relations between parameters which determine a repairman system.