Analysis of Reliability Characteristics Estimators in Accelerated Life Testing

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Abstract: The efficiency of nonparametric estimators with respect to parametric estimators for the accelerated failure time model is considered. This article gives the asymptotic properties of the parametric estimators and recall the properties of the nonparametric estimators proved by Bagdonavicius and Nikulin (1997).

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6.1 Introduction

Suppose that $S_{x(\cdot)}$ is the reliability function of the time-to-failure $T_{x(\cdot)}$ under the time varying stress $x : [0, \infty] \to B \subset \mathbb{R}^m$.

Consider the accelerated failure time (AFT) model [Bagdonavicius (1978)]:

$$S_{x(\cdot)}(t) = S_{x_0} \left\{ \int_0^t r(x(\tau))d\tau \right\},$$

(6.1)

where $S_{x_0}$ is the time to failure under the usual constant in time stress $x_0$, $r$ some positive function $r : B \to [0, \infty[$.

If $x(t) \equiv x = \text{const}$, then

$$S_x(t) = S_{x_0}\{r(x)t\}.$$

If the classical parametric and nonparametric estimation procedures [Nelson (1990), and Nikulin (1995)] the function $r$ is parametrized in the following way:

$$r(x(t)) = e^{\beta T_x(t)}$$
the function \( r \) is parametrized in the following way:

\[
r(x(t)) = e^{\beta^T z(t)}
\]

where \( r_0(x(t)) \equiv 1 \), \( \beta = (\beta_0, ..., \beta_m)^T \) is the vector of unknown parameters, \( z(t) = (z_0(t), ..., z_m(t))^T \) is the vector of some known functions of stress.

We consider the case when the function \( r \) is completely unknown, and the experiment is as follows:

Suppose that two groups of items are tested:

the first group of \( n_1 \) items is tested under the constant in time accelerated stress \( x_1 \) and the complete sample \( T_{11} \leq ... \leq T_{1n_1} \) are obtained;

the second group is tested under the stress

\[
x_2(\tau) = \begin{cases} 
  x_1, & \text{if } 0 \leq \tau \leq t_1, \\
  x_0, & \text{if } t_1 < \tau \leq t
\end{cases}
\]

and the type I censored sample \( T_{21} \leq ... \leq T_{2m_2} (m_2 \leq n_2) \) is obtained.

Such experiment is useful when variation coefficient of time-to-failure under the usual stress \( x_0 \) is not large and the most of failures occur in some interval \([\tau_1, \tau_2]\), where \( \tau_1 \) is larger than the time \( t \) given for experiment.

The items of the second group use much of their resource under the accelerated stress \( x_1 \) and after the moment \( t_1 \) even under the usual stress \( x_0 \) failures of items can be obtained (see, Figure 6.1).

![Figure 6.1: Test of two groups of items. The first one under constant stress \((x_1)\), and the second one under variable stress \((x_2)\).](image)