Curves in Polar Coordinates

Section 6.1 is a short introduction to using polar coordinates with MAPLE. In Sections 6.2–6.5 we plot the polar graphs of some remarkable curves (in particular, spirals, roses and crosses) and use inversion transformation.

In Chapter 6 the reader will become acquainted with the commands

polarplot, pieslice, conformal, coordplot, cot, inversion.

6.1 Basic Plots in Polar Coordinates

The location of a point \( M \) in the plane with origin \( O \) is uniquely defined by the distance \( |OM| = \rho \) and the angle \( \varphi \in [-\pi, \pi] \) between the segment \( OM \) and the polar axis. The real pair \((\rho, \varphi)\) is called the polar coordinates of the point \( M \).

The relation between polar coordinates \((\rho, \varphi)\) and rectangular coordinates \((x, y)\) when the axis \( OX \) plays the role of the polar axis is the following:

\[
x = \rho \cos \varphi, \quad y = \rho \sin \varphi
\]

These formulas were given by Isaac Newton in 1670. If \( \rho = \rho(\varphi) \) is the polar equation of a curve, then its equations in rectangular coordinates are

\[
x = \rho(t) \cos(t), \quad y = \rho(t) \sin(t).
\]

The conversion of a complex number \( z = a + ib \) into polar form is possible using the command \texttt{polar(number)}. The graph of \( \rho = f(t) \) in polar coordinates can be plotted using the command \texttt{polarplot} from the library \texttt{plots}. By assumption in MAPLE the parameter is given by \( t=-\Pi..\Pi \).
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> plots[polarplot](1, t=0..Pi);  # upper half-circle
> plots[polarplot](t, t=0..4*Pi);  # two coils of Archimedes’ spiral

The *spiral of Archimedes* $\rho = a\varphi$ (plotted above) was studied by Archimedes in the third century B.C. in relation to the problem of the trisection of an angle.

Also, plotting in polar coordinates is possible using the basic command `plot` with the additional option `coords=polar`. Hence the polygons and graphs of functions (see Figs. 6.1–6.2) in polar coordinates can be plotted analogously to the case of rectangular coordinates.

**Example 6.1.1**

1. We plot the polygon through some points of Archimedes’ spiral.

   > t:=i -> i*Pi/6: p1:=plot([seq([t(i), t(i)], i=0..40)], coords=polar, style=point, symbol=circle):
   > p2:=plot([seq([t(i), t(i)], i=0..40)], coords=polar):
   > plots[display]([p1, p2], scaling=constrained);  # Fig. 6.1

2. We plot a regular *star* $(m, n)$-gon (convex for $m = 1$) with relatively prime $m$ and $n$ (see also Example 17.3.1).

   > n:=5: m:=2:
   > plot([seq([1,m*i*2*Pi/n], i=0..n)], coords=polar);  # Fig. 6.2

   Using an easy generalization of this program, we plot the *disconnected star* $(m, n)$-gon.

   > n:=8: m:=2: t:=i -> i*2*Pi/n:
   > plot([seq([[1,t(i)],[1,t(i+m)]], i=1..n)], coords=polar);

3. We plot the *circular diagram* of Fig. 6.3.

   > with(plots): A:=[0, 10, 30, 40, 20]: # enter $A[2], A[3]...$ in %
   > B:=i->sum(A[j]/100)*2*Pi, j=2..i):
   > P:=polarplot([1, seq([[0,0],[1,B(i)]]], i=1..nops(A)-1)):
   > T:=textplot({seq([.5,B(i),convert('A'[i],string)], i=2..nops(A))}, coords=polar): display([P, T]);

   Another method is based on the command `pieslice` from the library `plottools`.

   > with(plots): P:=seq(display(plottools[pieslice]([0,0], 5, Pi*i/10..Pi*(i+1)/10, color=COLOR(HUE,evalf(i/20))), scaling=constrained), i=0..20): display({P},axes=none);

4. Let us plot the stopwatch with a *moving* arrow.

   > with(plots): n:=60:  # 60 seconds in a minute