Scaling, invariants and fixed points

*Abstract.* This brief chapter's goal is to show that, next to obvious and deep differences and without actual intent or interaction, fruitful and surprising parallels exist between my scientific work from 1956 to 1972 (as exemplified in M1997E and this book) and modern statistical physics.

The latter, to be denoted in this chapter as MSP, will be understood as focusing on critical phenomena. Pioneered in Kadanoff 1966, Wilson 1972 and Wilson & Fisher 1972, it was eminently successful in explaining or predicting many experimental observations. It started with solid and unquestioned laws of physics and used power-law relations and powerful tools called scaling and renormalization, invariance and fixed points.

In many important ways my work was very different. The early stage reported in M1997E concerned phenomena in economics, whose study cannot wait for the emergence of solid and unquestioned basic laws. As to the later work reported in this book, it concerned phenomena exemplified by turbulence. Solid and unquestioned basic laws are provided by the Navier-Stokes equations and there is a rich and effective phenomenology. Unfortunately, the link between those laws and that phenomenology remains elusive.

Granting those deep differences, two facts often come as a surprise. Firstly, my work used many of the same tools as MSP, namely, power-law relations and scaling and renormalization arguments. This happened long before the day in 1972 when I first heard of MSP and even before the period around 1965 when MSP gathered speed. Secondly, my use of those tools was extensive and (on its terms) successful. After 1972, I became increasingly influenced by MSP and worked on problems having a basis in solid and unquestioned laws. To a lesser extent, my work influenced MSP, and interesting further developments were influenced by both.

The main tradition that influenced me was the use of scaling and renormalization arguments in probability theory (as a fractured education
made me interpret that discipline). The most influential data were the power-law distributions discovered in the social sciences by Pareto and Zipf. A later influence was the use of scaling in the study of turbulence.

Particularly central to the point made in this chapter is the ancient use of power-law distributions and of scaling, fixed point, and renormalization arguments in the context of the Cauchy-Polyà-Lévy’s “stable probability distributions”. Those uses were purely mathematical until M1956c and M1956w injected them into a corner of science. They became central to the works I devoted from 1959 to 1972 to economics/finance (as reprinted in M1997E). They are also central to the works written before 1976 and reprinted in this book. Therefore, the deep connection that Jona-Lasinio 1975 discovered between the stable distributions and MSP automatically implies a deep connection between MSP and my work; this connection greatly contributes to this chapter’s thesis. A token of deep differences is that in MSP the parameters of the stable distributions follow from basic principles, while my work infers them from the data.

Inevitably, the inclusion of this brief special chapter is in part motivated by current efforts to expand the use of the methods of physics to domains where, despite long and systematic search, basic law are not available. A popular target domain is finance/economics. Every broadening is welcome, but finance had no need to borrow the basic ideas of scaling and renormalization from MSP, simply because my work had rooted them in finance even before MSP came to be.

THE HISTORICAL AND IN PART AUTOBIOGRAPHICAL TASK undertaken in Chapter N2 continues in this chapter, but in totally different style. Instead of many specialized topics, this chapter is restricted to a closely-knit complex of ideas surrounding power-law relations and probabilities distributions, scaling and renormalization, with an emphasis on invariances and fixed points. Instead of the activities of a few scholars working over a short time period, this chapter concerns events that engaged several large scientific communities over long time periods. A key factor is that those communities rarely interacted and could not view themselves as contributing to a common goal.

A broad comparative study of the various “flavors” of scaling and renormalization would be an excellent big project for a historian of science. It should include biological “allometry” and the long-known but also long-neglected scaling rules found in seismology. But the present