Chapter 3

Factor Decomposition of Economic Time Series Fluctuations – Economic and statistical models in harmony –

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3.1 Introduction

In analyzing factors of economic time series fluctuations, it is important to compare, first of all, the characteristics between a macro-econometric model and a time series model in order to explain to what extent an economic theory should be utilized or how deeply a statistical theory should be taken in. With the macro-econometric model, we make much of the fact that movement of a variety of factors mainly concerning business fluctuations should be described deterministically with the aid of the economic theory. At the same time, the macro-econometric model is often used for the procedures including simulation for the purpose of measuring the effect of the fiscal and monetary policies, and no attention is paid to the time series structure of economic variables. On the other hand, with a general time series model, we need none of economic theories to analyze the time series structure of the data. Therefore the model is described stochastically and we make much of the prediction performance.\(^1\) Needless to say, there might actually be neither an econometric model ignoring the time series characteristics nor economic time series analysis neglecting the economic theories. The purpose of this chapter resides in the improvement of the time series model by taking in the economic knowledge expressed by an econometric model in analyzing economic data.

Under recognition as described above, we try to make a model which expresses adequately the fluctuations of the real GDP as a representative macro-economic variable. The fluctuations of the real GDP can be grasped by classifying them into a long-term viewpoint and a short-term one. With the former, it is pointed out that the potential growth rate of the Japanese economy made drastic change in the first half of 1970's. On the other hand, the short-term fluctuations of the real GDP are divided into autonomous business fluctuations and economic policy oriented ones such as fiscal and monetary policy. We need a model expressing adequately such points as
described above. Accordingly the model in question should satisfy the condition shown below:

1) to express adequately the trend as the expression of the potential of the Japanese economy.
2) to express adequately the business fluctuations.
3) to measure the effect of the macro-economic policies such as discount rate operations.
4) to possess prediction ability.

In this chapter, in the beginning, we express the fluctuations of the real GDP by using a time series model, and when we can not interpret the obtained results, we use economic theory in order to improve the model. To grasp the fluctuations of the time series by dividing them into a long-term viewpoint and a short-term one, a state-space model proposed by Kitagawa and Gersch (1984) is to be utilized.

3.2 Model 1 (Model with Stochastic Components Only)

3.2.1 Outline of Model 1

Let \( Y_t \) be the real GDP (seasonally adjusted quarterly data, with the index taking up 1985 as 100, ranging from the first quarter of 1965 to the fourth quarter of 1991). Also let \( y_t \) be natural logarithm of \( Y_t \).

Considering that the real GDP is in stationary variation expressed as an autoregressive process around the stochastic trend, Model 1 is expressed as

\[ y_t = T_t + p_t + u_t, \quad \text{for} \quad t = 1, 2, ..., T. \]  \hspace{1cm} (3.1)

where \( T_t, p_t, \) and \( u_t \) represent the trend, autoregressive process, and observation noise, respectively.

Here, the trend component \( T_t \) is formulated by the model

\[ \Delta^m T_t = v_{T_t}, \quad v_{T_t} \sim N(0, \tau_T^2). \]  \hspace{1cm} (3.2)

On the other hand, the AR component \( p_t \) is formulated by

\[ p_t = a_1 p_{t-1} + \cdots + a_p p_{t-p} + v_{p_t}, \quad v_{p_t} \sim N(0, \tau_p^2). \]  \hspace{1cm} (3.3)

At that time, (3.1), (3.2) and (3.3) can be represented, as expressed in Kitagawa and Gersch (1984), with a state-space model

\[ x_t = F x_{t-1} + G v_t, \quad y_t = H x_t + u_t, \]  \hspace{1cm} (3.4)

where the state vector \( x_t \) is defined as

\[ x_t = (T_{t}, T_{t-1}, \ldots, T_{t-m+1}, p_{t}, p_{t-1}, \ldots, p_{t-\ell+1})^T. \]

Furthermore, \( F, G \) and \( H \) are given, for example in case that \( m = 2, \ell = 3 \), as shown below:

\[ F = \begin{bmatrix} 2 & 1 & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & a_1 & a_2 & a_3 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \end{bmatrix}. \]