2
Life Distributions, Models and Their Characteristics

2.1 Types of Failure Observations

A typical experiment in life testing of equipment consists of installing a sample of \( n \) similar units on appropriate devices and subjecting the units to operation under specified conditions until failure of the equipment is observed. We distinguish between two types of data. The first type is obtained under continuous monitoring of a unit until failure is observed. In this case we have exact information on the length of life, or time till failure, \( T \), of that unit. The observed random variable, \( T \), is a continuous variable, i.e., it can assume any value in a certain time interval. The second type of data arises when the units are observed only at discrete time points \( t_1, t_2, \ldots \). The number of failures among the \( n \) tested units is recorded for each inter-inspection time interval. Let \( N_1, N_2, \ldots \), denote the number of units failing in the time intervals \([0, t_1), [t_1, t_2), \ldots \). These are discrete random variables representing failure counts.

The proper analysis of data depends on the type of observations available. Experiments often must terminate before all units on test have failed. In such cases we have complete information on the time till failure (if monitoring is continuous) only on part of the sample. On all the units which have not failed we have only partial information. Such data are called time-censored. If all the units start operating at the same time we say that the censoring is single. Single time censoring is also called censoring of Type I. Some experiments terminate at the instance of the \( r \)-th failure, where \( r \) is a predetermined integer smaller than \( n \). In these cases the data are failure-censored. Single failure censoring is called censoring of Type II. If different units start operating at different time points in an interval \([0, t^*]\), and the experiment is terminated at \( t^* \), we have multiple censoring.
of data. We distinguish also between censoring on the left and censoring on the right. If some units began operating before the official time started we have left censoring. The other type of censored information, where the unit is still in operation at the termination of monitoring, is called right censoring.

2.2 General Characteristics of Life Distributions

We consider here the continuous random variable, $T$, which denotes the length of life, or the length of time till failure, in a continuous operation of the equipment. We denote by $F(t)$ the cumulative distribution function (CDF) of $T$, i.e.,

$$(2.2.1) \quad F(t) = \Pr\{T \leq t\}.$$ 

Obviously, $F(t) = 0$ for all $t \leq 0$. We assume here that initially the equipment is in good operating condition. Thus, we eliminate from consideration here defective or inoperative units. The CDF $F(t)$ is assumed to be continuous, satisfying the conditions

(i) $F(0) = 0$;
(ii) $\lim_{t \to \infty} F(t) = 1$;
(iii) if $t_1 < t_2$ then $F(t_1) \leq F(t_2)$.

**EXAMPLE 2.1**

The following is an example of a life distribution of a device which always fails between the time points $t_0$ and $t_1$, where $0 < t_0 < t_1 < \infty$:

$$F(t) = \begin{cases} 
0, & \text{if } t \leq t_0 \\
2 \left(\frac{t - t_0}{t_1 - t_0}\right)^2, & \text{if } t_0 \leq t \leq \frac{t_0 + t_1}{2} \\
1 - 2 \left(\frac{t_1 - t}{t_1 - t_0}\right)^2, & \text{if } \frac{t_0 + t_1}{2} \leq t \leq t_1 \\
1, & \text{if } t_1 \leq t.
\end{cases}$$

In Figure 2.1 we provide the graph of the life CDF, $F(t)$, for $t_0 = 100$ [hr] and $t_1 = 400$ [hr].

The reliability function, or the survival function of the equipment having a life length CDF $F(t)$, is defined as

$$(2.2.2) \quad R(t) = 1 - F(t), \quad 0 \leq t < \infty.$$ 

The reliability at time $t$ is the probability that the life length of the equipment exceeds $t$ [time units]. The survival function is the same as the reliability function.