STABILITY OF HIGH SPEED COMPRESSIBLE ROTATING COUETTE FLOW

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ABSTRACT

The linear stability of high speed couette flow is studied including full compressibility and viscosity effects for finite gap spacings between coaxial cylinders. Particularly, Mach number, Reynolds number, and radial heating effects on the stability of axisymmetric modes are investigated. Results are in complete agreement with the incompressible theory in the low Mach number limit. It is found that increasing Mach numbers have a stabilizing effect. Also, heating at the outer cylinder destabilizes and heating at the inner cylinder stabilizes the flow for all the Mach numbers covered in this work.

1. Introduction

Incompressible flow between concentric cylinders, goes through transition at much lower rotation rates when the inner cylinder rotates and the outer cylinder is at rest than vice versa (Mallock 1896, Couette 1890). According to Rayleigh's (1916) criterion, inviscid rotating flows will be stable if the square of the circulation decreases radially outward i.e. \( \frac{d}{dr}\left(r^2 v^2\right) < 0 \) where \( r \) is radius of curvature the flow is experiencing and \( v \) is tangential velocity. For symmetric disturbances about the axis, Rayleigh's criterion is a sufficient condition.

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Using the linear stability theory and including viscous effects, Taylor (1923) expanded on Rayleigh’s criterion and determined the condition for stability of couette flow to axisymmetric disturbances. He expressed the stability condition in closed form for small gap spacing between the cylinders and described the secondary flow which sets in after the first onset of instability as counter rotating cellular toroidal vortices of axial periodicity, i.e. the Taylor vortex flow. These structures have zero phase speed, therefore do not propagate. Further work on couette flow removed the narrow gap limitation (see Chandrasekhar 1961) and revealed that the first transition point is a bifurcation point for the time dependent Navier-Stokes equations (Kirchgassner & Sorge 1969). Accordingly, Taylor vortex flow is a second possible solution which exists besides the Couette flow. The non-uniqueness of the Navier-Stokes equations beyond the first critical point reveals an entire range of unstable axial wavenumbers, (Burkalte and Köschmieder 1974) the observed wavelength (and the cell pattern) of which depends on the time history of the flow. Snyder & Lambert (1966) investigated the non-linear aspects of this flow in their experiments.

For small gap spacing, the axisymmetric Taylor vortex flow itself becomes susceptible to azimuthal disturbances manifested as waviness in Taylor vortices. Beyond this second critical point, several unstable azimuthal wavenumbers appear on the spectrum. Depending on the path traveled in the parameter space, up to 20 or more states at a given speed were identified by Coles (1965).

For large gap spacing, this secondary instability is observed much later if it ever sets in and there is not enough experimental evidence showing that flow in the wide gap ever goes through the doubly periodic stage before final breakdown to transition. In this case, finite length effects have to be taken into consideration to truly simulate the flow (Street & Hussaini 1987) or before attempting to interpret the experimental findings (Snyder & Lambert 1966).

The effect of radial heating for the incompressible Taylor Couette flow can vary depending on whether one considers buoyancy forces or not. Roesner (1978) showed that with buoyancy, both heating and cooling stabilize the flow. According to Walovit, Tsao & Di Prima (1964) and Bahl (1972), Taylor vortices are stabilized by negative temperature gradients across the gap in the absence of buoyancy. Yih (1961) argued that the effects of viscosity and thermal diffusivity (separately or together) can be destabilizing. Viscosity is even capable of assuming a dual role which has been almost exclusively