Scaling arguments are applied directly to the Navier-Stokes equations with the isentropic-flow stipulation in conjunction with the scale-invariance condition on the rate of kinetic energy dissipation to derive the spectral law for the compressible isotropic turbulence when the random sound field is weak. For the special case with isothermal flow, the present result reduces to the Kadomstev-Petviashvili spectral law $\sim k^{-2}$. We will also address the nature of compressibility effects on the classical turbulent spectrum.

1. Introduction

The problem of scaling laws in a compressible isotropic turbulence has proved to be somewhat controversial since apparently conflicting views have been advanced by several workers. The first work on this subject was that of Zakharov and Sagdeev (1970) who investigated the spectrum of an isotropic random sound, and noting the fact that the three-wave resonant interactions for sound waves (which are nondispersive in the linear regime) occur when they have almost collinear wave vectors they derived a spectrum $\sim k^{-3/2}$ in the inertial range.

Another effect of compressibility is that energy can be continually radiated away in the form of sound waves (Lighthill 1955). Since this sound energy must ultimately be converted into heat by the various processes of acoustic attenuation, Moiseev et al. (1981) argued that compressibility effects act like a source of energy dissipation in addition to that provided by viscosity and thermal conductivity. By applying group-invariance principles to a Hopf-type functional formulation of the compressible case, Moiseev et al. (1981), accordingly, gave a spectrum steeper than the classical Kolmogorov spectrum for incompressible isotropic turbulence. However, the conditions for the validity of this spectral law were not clarified. Besides, their result is not completely
correct because it does not show the right dependence on the density and hence does not quite reduce to the classical Kolmogorov spectral law for incompressible turbulence in the appropriate limit.

As the nonlinear effects become prominent, the sound waves in a compressible fluid steepen to form shock waves; vortex formation behind the shock waves then produces shear turbulence. This has been confirmed by the laboratory experiments (Smits and Muck 1988 and Hesselink and Sturtevant 1988) as well as numerical simulations (Passot and Pouquet 1987 and Rotman 1991). These investigations further showed that the passage of a shock also results in smaller and more anisotropic eddies, with the fluid element being stretched in a direction parallel to the shock and compressed in the direction perpendicular to the shock. Kadomtsev and Petviashvili (1973) invoked the random orientation of the various shocks to justify the overall isotropy of the turbulence field despite the anisotropy of the individual shocks. Kadomtsev and Petviashvili (1973) then argued that all harmonics are phase-correlated together and damped so that the spectrum would have to be, however, steeper, and gave the spectral law $\sim k^{-2}$. Moiseev et al. (1977) argued, on the other hand, that the effect of the eddy viscosity of the random vortex field is to smooth out and dissipate the shocks so that the $k^{-2}$ spectrum would not have time to form. Elsasser and Schamel (1977) made numerical experiments and found, however, that their numerical solutions developed the shock spectrum $\sim k^{-2}$ instead of the weak-turbulence spectrum $\sim k^{-3/2}$.

The importance of scale-invariance principles in characterizing some essential features of complex phenomena in physics has never been in question. The raison d'être for the validity of these principles are certain symmetry properties possessed by the governing equations. We will show in the following that the scaling arguments applied directly to the Navier-Stokes equations with the isentropic-flow stipulation in conjunction with the scale-invariance condition on the rate of kinetic-energy dissipation yield the spectral law for the compressible isotropic turbulence when the random sound field is weak and correct an error in the corresponding result of Moiseev et al. (1981) and clarify the conditions for the validity of this result. For the special case with isothermal flow (considered by Kadomtsev and Petviashvili 1973), the present result reduces to the spectral law $\sim k^{-2}$ of Kadomtsev and Petviashvili (1973). This seems to provide a kind of scale-invariance rationale for the Kadomtsev-Petviashvili spectral law. We will also address the nature of compressibility effects on the classical turbulent spectrum.