5.1 Introduction

The beam is a very useful structural member that is employed in many different types of structural applications, such as floor, roof, and bridge deck systems. These members are called upon to resist bending action that is usually produced either by loads that are perpendicular to the member (transverse loads) or by pure bending moments. Regardless of the way the beam is loaded, the resulting action leads to two effects that are of major significance in the analysis of beams. These two effects are the induced stresses and the resulting deformations. Chapter 5 deals with the first of these two effects, the stresses induced in a beam, and the subject of deformations (or deflections) is dealt with in Chapter 6.

In order to discuss intelligently the question of stresses in beams, it is necessary to have a good understanding of properties of areas, including centroid location and principal centroidal moments of inertia, and their determinations. Thus, before discussing the concept of stresses in beams, it was decided to review briefly, in Section 5.2, the various concepts relating to properties of areas. This short review will ensure that the student has the background needed for proper understanding of the concepts of stresses due to symmetric and unsymmetric bending of beams.

5.2 Review of Properties of Areas

Consider the arbitrary plane area shown in Fig. 5.1 for which the centroid, point C, has been located. At point C, an xy coordinate system has been established as shown. Properties of the cross section with respect to axes through the centroid (i.e., centroidal axes) are examined in this section.

By definition, the moments of inertia* with respect to the centroidal coordinate x and y axes in Fig. 5.1 are given by

\[
\begin{align*}
I_x &= \int y^2 \, dA \\
I_y &= \int x^2 \, dA
\end{align*}
\]  

(5.1a)

where \(I_x\) and \(I_y\) represent the centroidal moments of inertia with respect to the x and y axes, respectively. Thus, by definition, the moment of inertia of an area with respect to a

* Common usage favors the term moment of inertia. A plane area, of course, really has no "inertia" associated with it, and the term second moment of area is more technically correct.
given axis requires that an element of area $dA$ be multiplied by the square of its distance from this axis and the resulting product integrated over the entire area. Therefore, the unit for the moment of inertia is a length raised to the fourth power, such as in.$^4$ in the U.S. Customary system and m$^4$ in the SI system.

Another quantity that is of great significance is the polar moment of inertia. This quantity, defined with respect to a centroidal axis, is given by

\[
J_c = \int \rho^2 \, dA
\]

where $J_c$ is the centroidal polar moment of inertia and the quantity $\rho$ is defined in Fig. 5.1. By using the fact that $\rho^2 = x^2 + y^2$, we can express $J_c$ in terms of $I_x$ and $I_y$. Thus

\[
J_c = \int \rho^2 \, dA = \int (x^2 + y^2) \, dA = \int x^2 \, dA + \int y^2 \, dA = I_y + I_x
\]

Note that the concept of the polar moment of inertia has already been used in the analysis of torsional members discussed in Section 4.2.

The radii of gyration for a given area with respect to the centroidal $x$ and $y$ axes in Fig. 5.1 are defined by the equations

\[
r_x = \left( \frac{I_x}{A} \right)^{1/2}
\]

\[
r_y = \left( I_y / A \right)^{1/2}
\]