The Cube Duplication by Abū Jaʿfar in the Manner of Nicomedes

Translations

The text of the cube duplication by Abū Jaʿfar (here denoted as F) is taken from Bibl. nat. ms. arab. (Slane) 2457, no. 47 (f. 198v–199r). As the author asserts, the construction in its opening section is based on the method of Nicomedes, as reported by Eutocius; the latter had already been rendered into Arabic by Thābit ibn Qurra, as part of his translation of Archimedes’ Sphere and Cylinder and Eutocius’ commentary on that work. For comparison’s sake, I insert after this section of Abū Jaʿfar’s text the analogous portions of Eutocius’ text, first in the version of Thābit (taken from Escorial ms. 960, f. 30r–31r; denoted here as F2), then directly from the Greek (Archimedis Opera, ed. Heiberg, III, pp. 104–106; denoted as F3). In the following section of his text, Abū Jaʿfar shows how a method via hyperbola effects the neusis that Nicomedes works out via the conchoid; this may be compared with the auxiliary conic construction introduced in the angle trisections by Pappus and Thābit (see chaps. B and C). Abū Jaʿfar closes with the demonstration of the cube duplication; I here display this in parallel with the comparable portion of the Arabic version of Eutocius (F2), differing in only trivial respects from the Greek (F3).

[From Abū Jaʿfar]

198v

(F) 1In the name of Allah the Merciful, the Compassionate. On the construction of two lines between two lines so that <all of them> follow in proportion, according to the method of fixed geometry, 2by the Shaik Abū Jaʿfar Muḥammad ibn al-Ḥusain, may Allah have mercy on him.

3Eutocius has cited in his book in which he collected writings of the ancient times from the masters of geometry on the construction of two lines between two known lines <so that all of them> follow in proportion, which Nicomedes devised in the construction of that according to the method of 5instrument, and we ourselves
describe the proposition which he has posited on this and has proved by it, except that we improve this to the method of geometry. 6And this is the manner.

Nicomedes posited the two lines \( AB, BG \) and made \( AB \) greater than \( BG \) and erected \( \overline{AB} \) to the other at a right angle, equal to angle \( B \). And he completed the figure \( ABGD \) and halved \( AB, BG \) at the two points \( E, Z \) and joined \( DE \) and extended it and \( GB \) so that they have met at \( H \), and \( BH \) has become equal to \( BG \). Then he drew a perpendicular \( \overline{ZT} \) without bound and from point \( G \) (he drew) the line \( \overline{GT} \) which equals each of the (lines) \( AE, EB \), and joined \( TH \) and drew \( GK \) without bound parallel to \( TH \); and he added \( GL \) to \( BG \) without bound. Then he persisted with the instrument that he invented until he drew a line from \( T \) to \( GL \) whose segment falling between \( GK \) and \( GL \) is equal to line \( GT \).

Figure F1. Cube-duplication by Abu Ja'far

[F2] Then since we have proved these clear things, then we make two known lines between which we wish to find two lines (continuously) proportional to them so that all of them follow in proportion, (these being) the two lines \( GL, AL \), and let each one of them be erect on its side at right angles, and let us divide the diagram \( \triangle ABGL \), parallel of sides, and let us divide each of the lines \( AB, BG \) in halves at the points \( D, E \), and let us join the line \( LD \) and extend it until it meets \( GB \), when

[F3] These things having been proved, let there be given the two lines \( GL, LA \) at right angles to each other, of which one must find two means continuously in proportion, and let the parallelogram \( ABGL \) be completed, and let each of the (lines) \( AB, BG \) be cut in half at the points \( D, E \), and the (line) \( DL \) having been joined, let it be extended and fall on \( GB \) extended at \( H \), and to \( BG \) (let there be drawn) at right (angles) the (line) \( EZ \), and let \( GZ \) be applied against it, equal to \( AD \), and