CHAPTER 10
The Anonymous Tract On Isoperimetric Figures

The Introduction to the Syntaxis, transmitted as an anonymous compilation of edited extracts from commentaries on Ptolemy's Book I, has drawn our attention already at three points. In dealing with its section on compound ratio, we have contested Mogenet's thesis that the Introduction was composed by Eutocius, and have argued that it be assigned instead to a commentator named Arcadius, active in the mid-5th century (see Part I, chap. 7). The Introduction's chapter on isoperimetric figures preserves certain testimonia on Archimedes' treatment of the circle, which, when coordinated with the witness of Pappus and Theon, contribute to the argument for the reconstruction of DC*, the lost prototype of the extant ancient and medieval texts of the Dimension of the Circle (chaps. 1 and 3). Moreover, the same chapter on isoperimetrics betrays interesting parallels with the medieval Latin De curvis superficiebus, and so supports the case for the lost prototype CS* (chap. 8). Another section of the Introduction, its account of long division, will concern us in the next chapter. We turn now to a more systematic examination of its account of isoperimetric figures.

This chapter of the Introduction is one of three complete expositions of isoperimetric figures that survive from antiquity. H.L.L. Busard has edited its Latin translation, De figuris isoperimetris, a literal Graeco-Latin rendering made in the 12th century. But no critical edition is yet available of the Greek, so that I will use the text prepared by F. Hultsch, in the supplement to his edition of Pappus, as the basis of my remarks. Although we have a reasonable notion of the date and provenance of the Introduction itself, as just said, it does not follow that the same is true for this chapter of it. For the limitations of expertise of the editor of the Introduction are so manifest, as in his stumbling account of compound ratio and other elementary arithmetic topics, that it is unlikely he is responsible in any significant way for the rather skillful rendition of the geometry of isoperimetric figures, a considerably more advanced topic than any other in his compilation; furthermore, the expansive commentatorial style of the Introduction contrasts
conspicuously with the austere exposition of the isoperimetric materials. I thus infer that this chapter represents the form of an earlier source on isoperimetric figures which the editor has incorporated virtually whole and minimally altered into his compilation. It is clear that this inference, which for the stated reasons I feel compelled to make, will complicate the project of delineating the source relations for this isoperimetric text. In the following I will treat it as anonymous, and, accordingly, designate it as AI.

A second account, given by Pappus, constitutes the first of the three principal sections of Book V of the Collection. We have already considered one portion of this account, its treatment of isoperimetric circular segments (props. 11–17), which appears only in Pappus' version (see chap. 5). I will designate it PI.

The third version appears in Theon's Commentary on Ptolemy’s Book I, where it explicates Ptolemy's allusion to the isoperimetric property of the circle and sphere as a metaphysical ground for the sphericity of the cosmos and the celestial motions. Ptolemy notes that these figures have special properties that ideally suit them to be templates for the world system; in particular,

of the different figures having equal perimeter, since the more polygonal are greater, of the plane figures the circle is greater, while of the solid ones the sphere <is greater>. (Syntaxis I 3)

Theon proposes to explain this remark by inserting an entire tract on isoperimetric figures:

We shall then make the proof of these things in epitome [en epitomêi] from the things proved by Zenodorus in the (book) On isoperimetric figures.

This is the only appearance of Zenodorus' name anywhere in the three isoperimetric tracts extant from antiquity. This fact has led Hultsch to the careless equation of Theon's account with the writing by Zenodorus, as if to suppose that Theon drew directly from Zenodorus and transmitted a more or less literal transcript of it. Furthermore, since in Coll. VIII (prop. 22) Pappus cites a proof of Archimedes' circle theorem that Pappus himself has given in his Commentary on Ptolemy's Book I, and since this very proof appears within the isoperimetric materials transmitted by Theon in his Commentary on Book I, Hultsch infers that the latter is in fact Pappus' commentary, but incorrectly attributed to Theon. Rome takes note of a similar passage where in his Commentary on Book VI Pappus refers to the same Archimedean proof given in the Commentary on Book I that has been interpolated into the Collection. In the light of Rome's efforts on Theon's commentary, Hultsch's surmise, that the Theonine attribution is false, can be dismissed as preposterous. But Rome's view is not a probable alternative, either, since it must assign both to Pappus' interpolator and to Theon the independent insertions of the same Archimedean