Among the axioms upon which the thermodynamics of continuous bodies is based in Lecture 2, the equation of balance of energy (1.7) and the Clausius-Duhem inequality (2.50) play the most important roles. In postulating (1.7) and (2.50) one necessarily has to treat the energy, entropy, and the absolute temperature occurring in these relations as primitive quantities whose meaning is clarified only through their consequences. Although this procedure is completely satisfactory from the logical point of view, the traditional pattern of presenting thermodynamics is different: energy, entropy, and absolute temperature are claimed to be derived from verbal statements of the "laws of thermodynamics". The arguments given in textbooks do not meet standards of rigor, but the program of founding thermodynamics upon postulates conceptually simpler than (1.7) and (2.50) is appealing. In this appendix we outline an approach to thermodynamics in which the existence of energy, entropy, and absolute temperature is rigorously proved.

Consider a cyclic process\(^2\) of the body \(\mathcal{B}\) in the period \([t_1, t_2]\), \(t_1 < t_2\). Accepting for a moment the existence of the energy and entropy and as-

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1 Here "conceptually simpler" means that fewer primitive concepts are involved but unfortunately it does not mean "easier to state".

2 A cyclic process is a process in which the body returns at the end to the initial state. We do not formalize the concept of state here, but mention that it has been basic to nearly all of the theoretical studies of the behavior of continuous bodies since the appearance of Noll's memoir "A new mathematical theory of simple materials" [1].

C. Truesdell, Rational Thermodynamics
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assuming that their present values are completely determined by the present state of the body, we see that necessarily

$$\varepsilon(x, t_1) = \varepsilon(x, t_2), \quad \eta(x, t_1) = \eta(x, t_2)$$

(G7.1)

and consequently

$$\mathcal{E}(t_1) = \mathcal{E}(t_2), \quad \mathcal{H}(t_1) = \mathcal{H}(t_2).$$

(G7.2)

Integrating the equation of balance of energy (1.7) over \([t_1, t_2]\) and taking into account (G7.2) yields

$$L = C,$$  

(G7.3)

where

$$L = \int_{t_1}^{t_2} \mathcal{W} \, dt$$  

(G7.4)

is the work done by the body in the process and

$$C = \int_{t_1}^{t_2} \mathcal{Q} \, dt$$  

(G7.5)

is the net gain of heat of the body in the process. The equation (G7.3), or its simple generalization

$$L = J C,$$  

(G7.6)

where \(J > 0\) is a universal constant independent of the process and of the body in question, expresses the universal interconvertibility of work and heat in cyclic processes. The constant \(J\) is called the mechanical equivalent of a unit of heat; with a suitable choice of units, \(J = 1\).

Integrating the Clausius-Duhem inequality (2.50) over \([t_1, t_2]\) and taking into account (G7.2) yields the Clausius inequality for cyclic processes

$$\int_{t_1}^{t_2} \left[ \int_{\partial \mathcal{B}} \mathcal{Q} \, dA + \int_{\mathcal{B}} \mathcal{S} \, dM \right] \, dt \leq 0,$$  

(G7.7)

where \(\mathcal{O}\) denotes the reciprocal of the absolute temperature \(\theta\), \(\mathcal{O} = 1/\theta\).

Below we shall give several statements concerning the properties of cyclic processes which under realistic additional assumptions imply the existence of a mechanical equivalent of a unit of heat \(J\) satisfying (G7.6) and the existence of an absolute temperature scale satisfying (G7.7). Some of these statements can be viewed as rigorous mathematical counterparts

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3 We use the notation of Lectures 1 and 2.

4 The attribution of (G7.7) to CLAUSIUS and likewise the attributions of the statements given below to particular persons should be understood in the sense of Footnote 4 of Lecture 2.