LECTURE 3

COLEMAN'S THEOREM

The principle of determinism, which is one of the main metaphysical ideas underlying all of natural science, asserts that past and present experience determines present response. In this generality and vagueness, it affirms the belief that such a thing as natural science is possible, and so we have only to look for it. In the construction of a particular branch of natural science we set up a mathematical model in which certain particular variables are made to represent the past and present, while others represent what it is they determine.

If we let \( f^t \) denote the history of the function \( f \) up to the time \( t \):

\[
f^t(s) \equiv f(t - s), \quad 0 \leq s < \infty,
\]

then a typical expression of the principle of determinism is

\[
g(t) = f(f^t),
\]

in which \( f \) denotes a functional, that is, a function whose arguments are functions. This equation states that if the history of \( f \) up to time \( t \) is known, the value of \( g(t) \) of \( g \) at time \( t \) is determined. In the more popular words favored by philosophers, \( f^t \) is the "cause", and \( g \) is the "effect". Throughout mathematical physics various special principles of determinism, involving the histories of various specific functions, are laid down as postulated constitutive relations and made the basis of mathematical development. All the theories of "classical" physics, whether old or new, are of this kind.

In the preceding lecture we have set up a substructure on which a thermodynamics of continuous bodies may be built. There a "cause" is the history of a thermokinetic process: \( x^t_k, \theta^t \), while its "effects" are the fields of free energetic \( \psi \), caloric \( \eta \), stress \( T \), and heating flux \( h \). As shown at the end of that lecture, the effective constitutive relations of thermodynamics are

\[
\begin{align*}
\psi(t) &= \mathbb{F}(x^t_k, \theta^t), \\
\tau(t) &= \mathbb{U}(x^t_k, \theta^t), \\
u(t) &= \mathbb{U}(x^t_k, \theta^t),
\end{align*}
\]

in which the argument \( X \), denoting the particle, is understood to enter both sides of these relations without being written there\(^1\), and for theories

\(\footnote{On the right-hand sides the unwritten argument is really all \( Z \) in a neighborhood of \( X \) rather than merely \( X \) itself.}\)

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of that kind the reduced dissipation inequality has the form

\[ \dot{\psi} - \tau \cdot \dot{\lambda} + \mathbf{i} \cdot \gamma \leq 0. \tag{2.62} \]

We repeat here the definitions of the quantities used:

\[ \tau \equiv \left( \frac{T(F^{-1})^T}{\rho}, -\eta \right), \tag{2.60} \]

\[ \mathbf{i} \equiv \frac{\hbar}{\rho}, \tag{2.63} \]

\[ \lambda \equiv (F, \theta) = (\text{Grad } x_\kappa, \theta), \tag{2.58}, \tag{2.60} \]

\[ \gamma \equiv -\frac{1}{\theta} \text{grad } \theta. \tag{2.45} \]

So as to shorten later statements, we shall call \( \tau \) the tension and the ordered pair \((\lambda, \gamma)\) the site. (The traditional term in thermodynamics is the "state", but use of it would permit the professional thermodynamicists to interrupt here with palaver about what "defines" a state, whether a state "exists", etc., etc.) The site at \( X \), of course, is determined trivially from the thermokinetic process in a neighborhood of \( X \). In order to be constitutive functionals, \( F, T, \) and \( \mathbb{I} \) must convert the history of an arbitrary thermokinetic process into a calorodynamic process. That is, the functionals \( F, T, \) and \( \mathbb{I} \) must be such that the reduced dissipation inequality (2.62) is satisfied identically in the thermokinetic processes \( x_\kappa(\cdot), \theta(\cdot) \).

The class of possible constitutive relations (2.64) is extremely general, far too general for us to be able to expect to get from it a thermodynamic theory in the classical sense, suggested by the properties of caloric "equations of state" (Lecture 1) and nineteenth-century ideas on irreversible processes (Lecture 2). Such a thermodynamic theory ought to have the following properties:

1. The free-energetic functional \( F \) is a thermodynamic potential in the sense that it determines uniquely the tension functional \( T \). (By (2.61), it follows that the functional \( \mathbb{D} \), whose value is the internal dissipation \( \delta \), is determined likewise by the free-energetic functional \( F \).)

2. The equilibrium response functions are independent of the temperature gradient and obey the classical potential relations of thermostatics (essentially (1.23)).

3. The internal dissipation determined by a thermokinetic process ending at a site of possible equilibrium is never negative.\(^2\)

\(^2\) This condition is weaker than the Planck inequality (2.27), which demands that \( \delta \geq 0 \) without exception. Nevertheless, the weaker inequality suffices to prove the theorem on isothermal cyclic processes (page 116, above), since the proof of the theorem requires the inequality (2.42) only for processes passing through possible equilibrium sites.