The laws of wave propagation cast light upon the nature of material response by analysis. They show how the material reacts, locally and instantaneously, to a small change or impulse in a tiny region. To compose the small effects into a motion of the body as a whole is a much harder problem even in the simplest of theories, a problem generally too hard to solve, yet we gain insight and assurance by taking the preliminary step even though we rarely follow through. The term “small” has two distinct meanings: that the disturbance itself is small, and that it is confined to a small region. There are several different approaches to wave motion, resting upon different concepts of smallness. In the commonest of these, the differential equations of motion are shorn of their non-linear terms so as to yield a linear system which may be visualized as an assembly of harmonic oscillators, whose motions may be described in the terms allowed by centuries of contemplation of the pendulum: frequency, amplitude, wave length, phase shift. It was to this method that HUGONIOT referred when, in 1885, he wrote: “... hypotheses have been imposed upon the equations of hydrodynamics which are disguised, it is true, by the word of approximation, but which singularly alter the value of such results as can be deduced from them.” HUGONIOT himself developed in fairly general terms a different concept of wave propagation in which the disturbance is limited, rigorously, to a region of no volume at all, namely, a surface, but the disturbance itself may be of any amount. The resulting theory of propagating singular surfaces is mathematically exact. By its aid, HUGONIOT himself was the first to give fully satisfactory calculations of the speeds of sound in gases and infinitesimally elastic solids. The values of these speeds agree with those gotten earlier by the methods HUGONIOT criticized with a harshness which today would not be tolerated in a young captain of naval artillery whose only other substantial publication, entitled “Study of the effects of powder in a 10-cm cannon,” had appeared just the year before. Apart from rigor, the difference in concept is great. While in the method of small oscillations the surfaces of disturbance are assumed to have a special form, usually plane, and the disturbances themselves depart little from equilibrium, the method of singular surfaces allows the discontinuity to lie upon a smooth surface of any shape, and the condition of the material may be any we please. Thus, for example, the conclusion for gases, \( \textit{viz} \)

\[ U^2 = p'(\rho), \]  

(4.1)

C. Truesdell, \textit{Rational Thermodynamics}  
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while of the same form, has different meanings in the two realizations. In the method of small oscillations, \( \rho \) is the density of the undisturbed gas, and \( p'(\rho) \) is therefore a constant: Each part of the wave front, which is generally assumed plane, advances at the same speed in the gas otherwise at rest. Thus the constant \( U \) is the speed of sound in classical, linear acoustics. The method of singular surfaces, on the contrary, envisions an arbitrary state of gas flow, and \( p'(\rho) \) is a function of the density, which in general varies over the flow field. Thus the function \( U \) is the speed of sound in gas dynamics, where the parts of a wave front may differ in swiftness of advance since they may pass through regions of different densities.

The method of HUGONIOT was developed by HADAMARD in his great treatise on wave motions, published in 1903, and it has always been known to specialists in mathematical gas dynamics. While it was presented in some books on physics fifty years ago, it then dropped out of the literature. With the rise of the new continuum mechanics it has come back into general use, especially in the last five years, because of its power in getting specific, concrete facts out of theories so inclusive as seemingly to defy analysis. In this lecture I shall outline the method and apply it to simple materials.

Let \( \Psi \) be a field defined and continuously differentiable in two regions \( \mathcal{R}_+ \) and \( \mathcal{R}_- \), of which a smooth orientable surface \( \mathcal{S} \) is the common boundary (Figure 6). Assume that at each point \( x \) on \( \mathcal{S} \) the field \( \Psi(y) \) approaches definite limits \( \Psi^+(x) \) and \( \Psi^-(x) \) as \( y \to x \) along paths lying entirely in \( \mathcal{R}_+ \) and \( \mathcal{R}_- \), respectively. At \( x \) itself the value \( \Psi(y) \) need not exist. The jump \( [\Psi](x) \) of \( \Psi \) at \( x \) is the difference of the two limits at \( x \):

\[
[\Psi] = \Psi^+ - \Psi^-.
\]

Assume further that the limit functions \( \Psi^+(x) \) and \( \Psi^-(x) \) are differentiable functions of position on the surface \( \mathcal{S} \). Then \( [\Psi] \) also is differentiable on \( \mathcal{S} \). If \( [\Psi] \) does not vanish at all points of \( \mathcal{S} \), the surface \( \mathcal{S} \) is said to be singular with respect to \( \Psi \). A singular surface, then, is the carrier of a jump discontinuity which is spread out smoothly over it. The field \( \Psi \) may be a scalar, vector, or tensor.

![Figure 6. Singular surface.](image-url)