5. The Bažant Method Based on the Effective Modulus

Sometimes, it is still common practice to consider the effects of creep by using a reduced modulus $E_{\text{eff}} = E_c/(1 + \varphi)$ to take into account creep effects. Since in this way the permanent deformations are rated equal to the elastic ones, the results were unsatisfactory initially. Bažant [12] suggested an improvement, analogous to Trost, by adapting, through correction factors, the value of the effective modulus $E''$ to the respective requirements of the relevant differential and/or integral equations.

Some excerpts from the work of Bažant [12] are included in the following.

... Then $f(t)$ varies linearly with $E_R(t, t_0)$ (relaxation function) and the stress-strain relations may be written in the form of an incremental elastic law:

$$\Delta f(t) = E''(t, t_0)[\Delta \varepsilon(t) - \Delta \varepsilon''(t)] \quad (5.1)$$

in which

$$\Delta \varepsilon(t) = \varepsilon(t) - \varepsilon(t_0), \quad (5.2)$$

$$\Delta f(t) = f(t) - f(t_0), \quad (5.3)$$

$$\Delta \varepsilon''(t) = \frac{f(t_0)}{E(t_0)} \varphi(t, t_0) + \varepsilon^0(t) - \varepsilon^0(t_0), \quad (5.4)$$

$$E''(t, t_0) = \frac{E(t_0)}{1 + \chi(t, t_0) \varphi(t, t_0)}, \quad (5.5)$$

$$\chi(t, t_0) = \left[1 - \frac{E_R(t, t_0)}{E(t_0)}\right]^{-1} - \frac{1}{\varphi(t, t_0)}, \quad (5.6)$$
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where \( \chi(t, t_0) \), \( E''(t, t_0) \), and \( \Delta \varepsilon''(t) \) will be termed aging coefficient, age-adjusted effective modulus, and fictitious inelastic strain increment.

Determination of \( \chi \) requires the knowledge of the relaxation function, which can be obtained from the creep function \( J_e(t, t') \) with the help of a computer. Table 5.1 shows the values of \( \chi \) which have been found for the following material properties:

\[
\varphi(t, t') = \varphi_a(t') \frac{(t - t')^{0.6}}{10 + (t - t')^{0.6}}
\]

(5.7)

or

\[
\varphi(t, t') = \varphi_a(t')0.113 \ln(1 + t - t'),
\]

(5.8)

where

\[
E(t') = E(28) \left( \frac{t'}{4 + 0.85t'} \right)^{1/2},
\]

\[
\varphi_a(t') = \varphi(\infty, 7)1.25t^{-0.118},
\]

(5.9)

where \( t, t' \) is the time from casting of concrete being given in days and \( t_0 \) is the time at first load application.

Equation (5.7) through Eq. (5.9) have been recently recommended by ACI Committee 209, along with a method of determination of the constant \( \varphi(\infty, 7) \). Equation (5.7) is acceptable for structures of normal dimensions that are exposed to a mild climate and allowed to dry. Equation (5.8) is suitable for mass concrete.

For the purpose of comparison, the \( \chi \) values have also been computed for creep functions Eq. (5.7) and (5.8) with a constant modulus \( E_c \) (Table 5.1)."

We can see that for the normal case (normal creep coefficient, normal load age, and duration of loading), the aging coefficients \( \chi \) vary little, even for a variable modulus of elasticity. This also follows from the works of Trost [10] and Schade [18]. The mean value is approximately 0.8.

In conclusion, it has to be said that the relaxation coefficient \( \varphi \) according to Trost is of the same importance as the aging coefficient \( \chi \) according to Bažant. The numerical value of \( \chi \) and \( \varphi \) should be the same. Equation (4.1), as written with the age-adjusted effective modulus, will read as follows:

\[
\dot{\varepsilon}_t = \frac{1}{E_c} f_0(1 + \varphi_t) + \frac{1}{E''} (f_t - f_0) + \varepsilon_{st},
\]

where

\[
E'' = \frac{E_c}{1 + \rho \varphi_t}.
\]