ON CONTINUOUS TIME LEARNING MODELS

by

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1. Inter-occurrence times of learning events

Learning models (or random systems with complete connections) like the Bush-Mosteller-type models evolves on two levels, the level of (unobservable, hypothetical) states $w \in W$, $(W, W)$ being the state space, and on the level of (observable) events $i \in I$, where the event space

$I = \{1, 2, \ldots, m\}$

will be supposed to be finite. The step by step transitions are governed by a transition probability $P$ from $W$ to $I$ and by a measurable state transformation $u$ from $W \times I$ to $W$, as illustrated in Fig.1. If a starting element $w^{(0)} \in W$ is given the model can be described by two sequences $w^{(n)}$ and $\xi^{(n)}$, $n \geq 1$, of random variables on a probability space $(\Omega, \mathcal{F}, P)$, with values in $W$ and $I$, respectively (see [4] Th. 2.1., p. 64 and Prop. 2.1.4, p. 66).

As an example consider a linear learning model, namely the so-called linear OM-chain (Onicescu and Mihoc [8]), where $W$ is the set of probability vectors

$w = (w_1, \ldots, w_m)$

and where

$P(w, i) = w_i$

$u(w, i)_j = a_i w_j + (1 - a_i) \Lambda_{ij}$

with coefficients $a_i$, $0 < a_i < 1$ and with an $m \times m$ stochastic matrix $\Lambda = (\Lambda_{ij})$. If all $a_i$ are zero, we are faced with a simple Markov chain (see [9] for a detailed analysis of linear OM-chains).

Up to now events are supposed to occur on a discrete time scale. In order to implement information on the inter-occurrence times, Iosifescu [3] introduced a third sequence $\eta_n$, $n > 0$, of $\mathbb{R}_+$-valued random variables on $(\Omega, \mathcal{F}, P)$, the waiting times or interoccurrence times between two succeeding events. Now the evolution of the process is governed by a transition probability $\mathcal{P}$ from $W$ to $X$, where

$X = \mathbb{R}_+ \times I$, 

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Evolution of a discrete time learning model (or random system with complete connections) on the two levels of states and events, respectively.

Fig. 1. Evolution of a learning model (or random system with complete connections) when inter-occurrence times are implemented.

Fig. 2. Evolution of a learning model (or random system with complete connections) when inter-occurrence times are implemented.